



education

Department:
Education
PROVINCE OF KWAZULU-NATAL

**JUST IN TIME MATERIAL
KZN DEPARTMENT OF EDUCATION
CURRICULUM GRADES 10 – 12 DIRECTORATE
TERM 1 – 2017**

MATHEMATICS

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the content and give more guidance to the teachers.

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ALGEBRAIC EXPRESSIONS

ATP REFERENCE

DATES	CURRICULUM STATEMENT	% COMPLETED
11/01 – 16/01 (4 days)	1. Understand that real numbers can be rational or irrational. Know the difference as far as the decimal expansions of the numbers are concerned. 2. Establish between which two integers a given simple surd lies.	3%
17/01 – 23/01 (5 days)	3. Round decimal numbers to an appropriate degree of accuracy. 4. Multiplication of a binomial by a trinomial. 5. Factorization to include types taught in grade 9 and : <ul style="list-style-type: none"> • trinomials • grouping in pairs • sum and difference of two cubes 	6%
24/01 – 0/01 (5 days)	6. Simplification of algebraic fractions using Factorization	9%
31/01 – 03/01 (4 days)	7. Addition and subtraction of algebraic fractions With denominators of degree at most 3.	12%

MARCH COMMON TEST WEIGHTING

Algebra (All parts)	35±3 marks out of 50 marks
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CAPS EXAM GUIDELINE WEIGHTING FOR FINAL EXAMINATION

Algebra, Equations (and inequalities)	30 ± 3 marks out of 100marks
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Topic	ALGEBRA PART 1 (Algebraic Expressions)
Weighting	±2 Marks
Sub-topics/Clarification	Number System
Related concepts/terms/vocabulary	Real Numbers (Rational & Irrational). Non-Real numbers.
Prior-knowledge/ Background knowledge	Natural Numbers, Whole Numbers, Integers, Perfect square numbers, Square roots, Cubes and cube roots, common fractions, decimal fractions.
Resources	Calculator, Textbook.
Educator Activity	May ask questions to test prior knowledge May give a short written class exercise (peer or group work) to test prior knowledge. Give feedback. Teach terminating & non terminating decimals, recurring & non-recurring decimals as well as surds as a way of classifying rational and irrational numbers.
Learner Activities	Class exercise: Classify Numbers as Rational; Irrational or Neither, and then place real numbers on a number line. (Refer to annexure)
Methodology	Ask questions. Use a number line to illustrate the positioning of all the subsets of real numbers.
Errors and Misconceptions	Remind learners that $\frac{22}{7}$ is only an estimation of π . $\frac{22}{7}$ is rational whereas π is an irrational number.

	Lesson 2
Topic	Converting decimal fractions to common fractions
Weighting	±3 marks
Sub-topics/Clarification	Converting a recurring decimal to a common fraction; Rounding off to an appropriate degree of accuracy.
Related concepts/terms/vocabulary	Equations and variables
Prior-knowledge/ Background knowledge	Four Operations
Resources	Text book
Educator Activity	Teacher asks questions: Solve simple equations like $3x - x = 4$; Convert common fraction to decimal and discover pattern. Teacher will demonstrate with examples how to round off to an appropriate degree of accuracy. The educator will demonstrate the steps to follow in converting the recurring decimals to common fractions without using a calculator.
Learners activity	Will do an activity to convert the different types of recurring decimal fractions. Learners will do an activity to round off decimals to an appropriate degree of accuracy. (Refer to the annexure).
Assessment	Homework from textbook.

	Lesson 3
Topic	Establishing between which two consecutive integers a given surd lies. (no calculator)
Weighting	±3 marks
Sub-topics/Clarification	Perfect squares & square roots; Perfect cubes & cube roots; Integers.
Related concepts/terms/vocabulary	Number line
Prior-knowledge/ Background knowledge	Real Number system
Resources	Textbook
Educator Activity	Pre-knowledge (Question & Answer) Teacher demonstrates using a number line how to identify integers from perfect square numbers / perfect cubes to establish between which two consecutive integers a given surd lies.
Learners activity	Do an activity (Refer Annexure)
Assessment	Homework from textbook.
Errors/Misconceptions/ Problem Areas	Emphasize Consecutive integers.

	Lesson 4
Topic	Multiplication of Binomial by Trinomial
Weighting	±3 marks
Sub-topics/Clarification	Distributive law of multiplication
Related concepts/terms/vocabulary	Terminology: Constant; Coefficient; Variables; Like and unlike terms
Prior-knowledge/ Background knowledge	Polynomials; Laws of Exponents
Resources	Text book
Educator Activity	Recall of prior knowledge by giving a group activity on Polynomials. (you may use annexure) Initiate discussion on addition and subtraction of terms (based on learners knowing how to identify like and unlike terms). Teacher may give a discovery activity where learners will find products of : A monomial by a monomial; monomial by a binomial and binomial by binomial, with the purpose of discovering a binomial by a trinomial product.
Learners activity	Will do activities on polynomials. Learners will discuss. Learners will answer questions on monomial; binomial and trinomial products. (Refer to the annexure).
Assessment	Homework from textbook.
Errors/Misconceptions/ Problem Areas	1. $x + x \neq x^2$ also $3x^2 + 2x^2 \neq 5x^4$ (Emphasize laws of exponents) 2. $x - 3(x + 2)$ is not the same as $(x - 3)(x + 2)$

	Lesson 6
Topic	Simplification of algebraic fractions
Weighting	±6 marks
Sub-topics/Clarification	Multiplication and division of fractions
Related concepts/terms/vocabulary	Numerator; Denominator; factorisation
Prior-knowledge/ Background knowledge	Factorisation of algebraic expressions; Multiplication and division of common fractions.
Resources	Text book
Educator Activity	Teacher will teach learners the steps to follow in simplifying fractions, start with simple single term expressions then extend to complex algebraic expressions.
Learners activity	Learners will do exercises to master the skill of simplifying fractions. (Refer to the annexure).
Assessment	Homework from textbook.
Errors/Misconceptions/ Problem Areas	Cancel factors only if one is in the numerator and one is in the denominator.

	Lesson 7
Topic	Addition and subtraction of algebraic fractions with denominators of degree at most 3
Weighting	±8 marks
Sub-topics/Clarification	Addition and subtraction of algebraic fractions
Related concepts/terms/vocabulary	Lowest common denominator, terms.
Prior-knowledge/ Background knowledge	Factorising algebraic expressions; simplifying algebraic expressions; Adding and subtraction common fractions.
Resources	Text book
Educator Activity	Teach learners the steps to follow in the process of adding and subtracting fractions. Start with simple single term expressions, and extend to complex expressions.
Learners activity	Learners will do practice exercises to master the skill of adding and subtracting algebraic fraction (Refer to the annexure).
Assessment	Homework from textbook.
Errors/Misconceptions/ Problem Areas	

ACTIVITIES

1. Rewrite the following numbers as fractions and state whether each number is a rational or irrational.

$$\begin{array}{ccccccc}
 5 & -2 & 4,1 & 2\frac{1}{3} & 3,14 & 0,\dot{7} & \sqrt{9} \\
 -\sqrt[3]{8} & \sqrt{10} & \sqrt{1\frac{9}{16}} & \frac{0}{5} & \frac{5}{0} & \sqrt{-9} & \sqrt[3]{9}
 \end{array}$$

2. Complete the classification of numbers by writing a tick in the appropriate box.

	Non- real	Real	Irrational	Rational	Integer	N _o	N
5							
-2							
4,1							
$2\frac{1}{3}$							
3,14							
$\sqrt{9}$							
$\sqrt[3]{9}$							
$\sqrt{-9}$							
$\frac{5}{0}$							
$\frac{0}{5}$							
$\sqrt{10}$							
$-\sqrt[3]{8}$							

3. Convert each the following to decimals:

a) $\frac{1}{2}$

b) $1\frac{1}{3}$

c) $\frac{7}{11}$

4. Given the following set of numbers:

$$\frac{25}{20}$$

$$\sqrt[3]{-8}$$

$$\sqrt{\frac{6}{2}}$$

$$\frac{6}{0}$$

$$-\sqrt{\frac{9}{3}}$$

$$\sqrt{\frac{0}{2}}$$

$$\sqrt{-8}$$

$$\pi$$

$$4,\dot{1}$$

$$3,\dot{1}\dot{2}$$

4.1 Write down two irrational numbers.

4.2 Write down one non- real number.

4.3 Write down one undefined number.

4.4 Rearrange the real numbers from smallest to largest.

4.5 Draw a number line and show the positions of these numbers.

NB: For more activities see the attached annexure

EXPONENTS

ATP REFERENCE

06/02 –10/02 (5 days)	<p>1. Revise laws of exponents learnt in Grade 9 where $x, y > 0; m, n \in \mathbb{Z}$:</p> <ul style="list-style-type: none"> • $x^m \times x^n = x^{m+n}$ • $x^m \div x^n = x^{m-n}$ • $(x^m)^n = x^{mn}$ • $x^m \times y^m = (xy)^m$ <p>Also, by definition:</p> <ul style="list-style-type: none"> • $x^{-n} = \frac{1}{x^n}, x \neq 0$ and $x^0 = 1, x \neq 0$. 	15%
13/02 –17/02 (5 days)	<p>2. Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for $m, n \in \mathbb{Q}$.</p>	18%

MARCH COMMON TEST WEIGHTING

Exponents	15 marks out of 50 marks
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CAPS EXAM GUIDELINE WEIGHTING FOR FINAL EXAMINATION

Algebra Equations (and inequalities)	15 ± 3 marks out of 100marks
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Sub-topics/ Clarification	<ul style="list-style-type: none"> • Revise laws of exponents learnt in Grade 9 and definitions • Use the laws of exponents to simplify expressions and solve equations
Related concepts/ terms/vocabulary	<ul style="list-style-type: none"> • Exponential notation (power, base and index/ exponent). • Notation of negative exponents • Rational exponents
Prior knowledge/ Background knowledge	<ul style="list-style-type: none"> • Laws of exponents • Factorization • Addition/ subtraction of like terms • Prime numbers • Rational numbers • Number systems
Resources	Calculator
Educator Activities	<ul style="list-style-type: none"> • Revise laws of exponents learnt in grade 9 • Demonstrate how to solve rational exponents and the application of the laws of exponents • Demonstrate exponential expressions using factorization • Demonstrate solving exponential equations <p>NB: Difference between equation and expression.</p>
Learner Activities	<ul style="list-style-type: none"> • Expand powers • Apply laws of exponents to simplify problems • Solve exponential equations
Methodology	<ul style="list-style-type: none"> ➤ Use, for example, 5^2 to explain exponential notation (power, base and index or exponent). Then give learners a short exercise to check their understanding. ➤ Ask learners to give examples on expanding exponential expressions e.g $3^3 = 3 \times 3 \times 3$ or $x^2 = x \times x$ and then give a short exercise on this. Also define $a^n = a \times a \times a \times \dots \quad (\text{to } n \text{ factors})$ ➤ Explain the laws of exponents and provide examples: For $a > 0, b > 0$ and m and $n \in \mathbb{Z}$ $a^m \times a^n = a^{m+n}$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n \times b^n$$

$$(a^m)^n = a^{mn}$$

- Important definitions: $a^0 = 1$ $a \neq 0$
 $a^{-n} = \frac{1}{a^n}$ $a \neq 0$ and $n \in N$
 $a^{\frac{1}{n}} = \sqrt[n]{a^n}$ and
 $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $a \neq 0$ and $n \in N$ and $m \in Z$

Provide examples and exercises that need application of these definitions. With these definitions, all the laws of exponents also hold for rational exponents.

- Provide examples on application of laws of exponents:
- Multiplication and division of exponential expressions including rational and negative exponents.
 - Addition and subtraction of exponential expressions.
- Provide examples on how to solve exponential equations.

NB: Solve without using a calculator

QUESTIONS (Classroom Mathematics)

1. Given a^x :

(a) What do we call the expression above?

(b) What is 'a' called?

(c) What is 'x' called?

2. Write in expanded form:

(a) 1^5 (b) x^2 (c) $(ab)^4$ (d) ab^4 (e) 4 to the exponent of 5.

3. Calculate:

(a) 2^4 (b) $5^2 \times 2^2$ (c) $(5 \times 2)^2$ (d) $(5 + 2)^2$ (e) $5^2 + 2^2$ (f) $5^{-1} \times 2^{-2}$

4. Simplify and calculate:

(a) $2^3 \times 2$ (b) $3^2 \times 2^3$ (c) $2^3 + 2$ (d) $(p^2q^4)^3$ (e) $3(xy)^2$

(f) $(3xy)^2$ (g) $\frac{4x^2y^5}{2xy}$ (h) $\frac{6x^{-2}y^5}{3xy^2}$ (i) $3^0 \times 3^4$

(j) $5^2 \times 2^3 \times 2^4 \times 5^3$

(k) $ab^2 \times b \times b^7 \times a^5 \times a^6 b^0$ (l) $\frac{15x^6y^{-4}z^3}{20x^3y^4z^6}$ (m) $\frac{28p^0q^4}{4qp^6r} \div \frac{21p^{-4}q^3}{r^{-1}}$

(n) $\left(\frac{1}{a+b}\right)^{-2}$ (o) $\left[\frac{4}{2ab^{-1}}\right]^{-1}$ (p) $\left[\frac{ab}{2^{-3}}\right]^{-1}$

5. Simplify without using a calculator:

(a) $16^{\frac{1}{2}}$ (b) $(\frac{1}{27})^{\frac{1}{3}}$ (c) $8^{\frac{2}{3}} \cdot 16^{-\frac{1}{2}} \cdot 625^{\frac{1}{4}}$ (d) $(9 + 16)^{\frac{1}{2}}$

(e) $\frac{x^2 y^0 \times x^3 y^{-2}}{x y^{-1} \times x^4 y^{-1}}$ (f) $\frac{3^2 x^{-1} y^2 \times x^{-3} y^{-2}}{x \times 3 x^{-4}}$ (g) $\frac{6(xy)^2 \times 3^2 (xy)^{-3}}{18xy}$

(h) $\frac{10^3 \cdot 15^4 \cdot 6^2}{20^4 \cdot 45^2 \cdot 12}$ (i)

(k) $\frac{12^{x+1} \times 9^{2x-1}}{36^x \times 8^{1-x}}$

6. Simplify:

(a) $\frac{2^x - 2^{x-2}}{2^{x+1} - 2^x}$ (b) $\frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+2} \cdot 7}$ (c) $\frac{x^{-1} + y^{-1}}{x^{-1} y - y^{-1} x}$

(d) $\frac{5^{x+2014} + 5^{x+2012}}{5^{x+2013} + 5^{x+2011}}$ (e) $\frac{6(3^{m+1})}{(3^m)^{m-1}} \div \frac{2 \times 9^{m+1}}{(3^{m-1})^{m+1}}$ (f) $\frac{4^n \cdot 2^{n+2} - 8^n}{2^2 \cdot 3^0 \cdot 2^{3n}}$

7. Solve for x

7.1 $2^x = 16$

7.2 $x^{-2} = 25$

7.3 $\frac{1}{3^x} = 27^2$

7.4 $2 \cdot 3^x = 18$

7.5 $4^{2x} = 8^{3x-4}$

7.6 $3x^4 = 48$

7.7 $4^{-x} = 8$

7.8 $5^{x+1} = 1$

7.9 $3^m + 3^{m+2} = 90$

7.10 $\frac{3 \cdot 2^{2m+1} - 2^{2m+1}}{4^m} = 2^{-2m}$

EQUATIONS & INEQUALITIES

ATP REFERENCE

20/02 –24/02 (5 days)	1. Revise the solution of linear equations. 2. Solve quadratic equations (by factorization).	21%
20/02 –24/02 (5 days)	3. Solve simultaneous linear equations in two unknowns. 4. Solve word sums involving linear, quadratic or simultaneous linear equations.	24%
27/02 –03/03 (5 days)	5. Solve literal equations (changing the subject of a formula). 6. Solve linear inequalities (and show solution graphically). Interval notation must be known.	27%

MARCH COMMON TEST WEIGHTING

Equations and inequalities	35±3 marks out of 50 marks
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CAPS EXAM GUIDELINE WEIGHTING FOR FINAL EXAMINATION

Algebra Equations (and inequalities)	30 ± 3 marks out of 100marks
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1. Activities for linear equations (solve the following linear equations)

1.1 $x + 2 = 10$

1.2 $2x + 8 = 12$

1.3 $4a + 28 = 10 - 2a$

1.4 $3(2x + 4) = 4(x - 1)$

1.5 $\frac{a}{2} + \frac{a}{5} = 14$

2. Activities for quadratic equations (solve the following quadratic equations)

2.1 $x^2 + 4x + 3 = 0$

2.2 $2x^2 = 8$

2.3 $6x^2 + 7x = -2$

2.4 $(x - 5)(x - 7) = 3$

2.5 $3x^3 - 2x^2 - 8x = 0$

3. Activities on inequalities (solve the following inequality equations)

3.1 $7x - 5 > 3x + 11$

3.2 $-5x + 30 < 20$

3.3 $-3x \geq 9$

3.4 $5 < x + 3 \leq 9$

3.5 $-1 \leq 2x - 3 < 5$

3.6 $\frac{x-5}{3} - 2x + 10 < \frac{x-5}{2}$

4. Activities on simultaneous equations (solve for x and y)

4.1 $2x + 4y = 20$ and $7x - 3y = 19$

4.2 $3(x + y) + y = -8$ and $2x + y = -7$

4.3 $\frac{x}{2} + \frac{y}{3} = 10$ and $x + \frac{y}{2} = 16$

4.4 $9x + y = 40$ and $5x - 3y = 8$

5. Activities on lateral equations (change the subject in each of the formulas as shown below)

5.1 $V = IR$ $R = ?$

5.2 $v = u + at$ $a = ?$

$$5.3 \ E = mc^2 \quad c = ?$$

$$5.4 \ x = \frac{pM}{m+M} \quad M = ?$$

$$5.5 \ s = \frac{v+u}{2} \times t \quad u = ?$$

6. Mathematical modelling type question (word problems)

6.1 In a traditional wedding MR Sithole slaughtered chickens and cows, he had 13 heads and 30 legs for both chickens and cows, how many chickens and cows were slaughtered?

6.2 In a garage there is a total of 50 cars and motorbikes. The total number of wheels is 172. How many motor bikes are there?

6.3 If the number is halved and 3 is added, the result is 17. Find the number

6.4 If nine times a certain number is subtracted from 120, the result is 10 less than the number. Find the number.

6.5 The sum of three consecutive numbers is 24. Determine the numbers.

6.6 The ages of a mother and daughter total to 36 years. In seven years' time the mother will be four times as old as the daughter is then. How old are they now?

Exam type questions (solve for the unknowns)

1. $x^2 - 5x = 0$

2. $x^2 - x - 20 = 0$

3. $2x^2 + 18x + 36 = 0$

4. $8x^2 + 2x - 3 = 0$

5. $x^2 - \frac{1}{4} = 0$

6. $2x + y = 3$ and $x + 3y = -1$

7. $\frac{a}{2} + \frac{b}{3} = 10$ and $a + \frac{b}{2} = 16$

8. Solve the following linear inequalities

8.1 $3 < \frac{x}{2} - 1 \leq 4$ illustrate your answer on a number line

8.2 write the solution of 8.1 in interval notation

9. Solve the following by making the asked variable the subject of the formula

$$9.1 V = \pi r^2 h \quad h = ?$$

$$9.2 V = \frac{1}{3} \pi r^3 \quad r = ?$$

$$9.3 A = P(1 + in) \quad i = ?$$

$$9.4 E = \frac{1}{2} mv^2 \quad v = ?$$

NUMBER PATTERNS

ATP REFERENCE

13/02 –17/03 (5 days)	Patterns: Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear. (Do not use the formula $T_n = a + (n - 1)d$.)	30%
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MARCH COMMON TEST WEIGHTING

Number Patterns	15±3 marks out of 50 marks
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CAPS EXAM GUIDELINE WEIGHTING FOR FINAL EXAMINATION

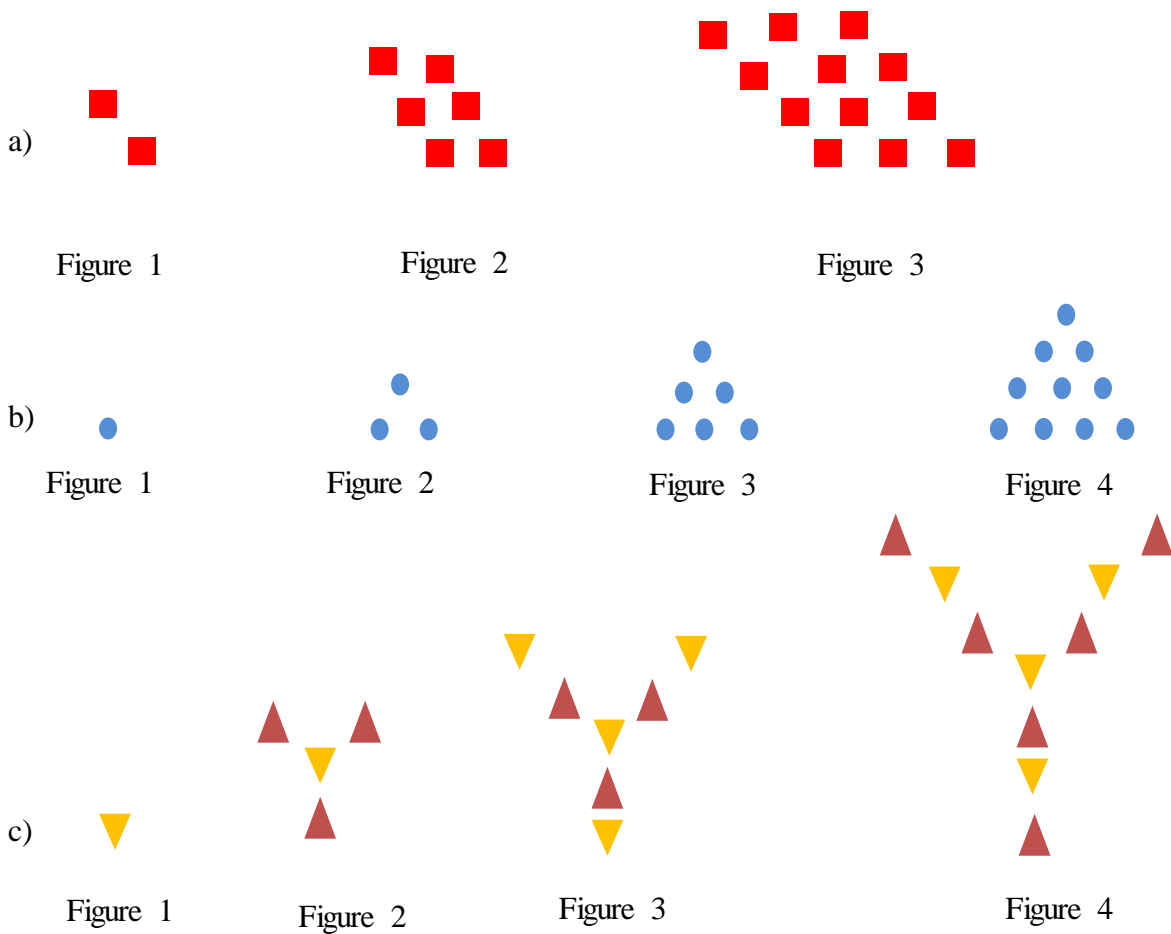
Number Patterns	15±3 marks out of 100marks
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Sub-topic/ Clarifications	Linear Number Patterns General term to be used $T_n = an + q$
Related concepts/ terms/ vocabulary	Term, Term number, Constant difference (a), Consecutive terms, n^{th} term (general term), conjecture
Prior knowledge/ background knowledge	Finding difference between terms, finding the next term, substitution, solving equations (linear equations)
Resources	Real objects (e.g marbles, dice, match sticks etc), calculator, worksheet
Educator Activity	Demonstrate using objects to identify the pattern
Learner Activities	Use given pattern/ sequence to develop more patterns/ sequences
Methodology	<ul style="list-style-type: none"> Investigate number patterns leading to those where there is constant difference between the consecutive terms, and the general term is therefore linear.

CLASS ACTIVITIES

ACTIVITY 1

1. Look at the figures below and create the next two figures



2. How many objects are making each of the figures above, a) to c)?

ACTIVITY 2

1. In each sequences below write down:

- (i) the next 3 terms
- (ii) the n^{th} term(T_n)
- (iii) the 100^{th} term

A) 1; 4; 7; 10;

B) 14; 8; 2;-4;

C) 23; 19; 15; 11; ...

D) -9; -5; -1; 3;.....

2. Study the given number pattern: $x + 3; 2x + 5; 3x + 7; \dots$

- a) Write down the next TWO terms in the sequence (in terms of x).
- b) Determine the general term (T_n).
- c) If the 10^{th} term has a value of 221, determine the value of x .
- d) Which term in the sequence has a value of 331?
- e) Is 440 a term in the sequence? Justify your answer.

3. Find the first TWO numbers in the sequence:

_____ ; _____ ; 4 ; 7; 12 ; 19; 28

4. Given the sequence: 6; 10; 14;

Show that the terms of the sequence are always even.

EXAM TYPE QUESTIONS

Activity 1- March 2013

Question 1

$1-p$; $2p-3$; $p+5$; are the first three terms of a linear number pattern

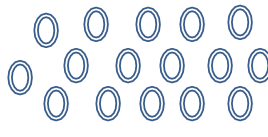
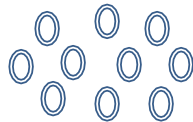
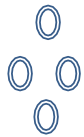
- 1.2 If the value of p is three, write down the FIRST THREE terms (3)
- 1.3 Determine the formula for the T_n , the general term of the sequence (2)
- 1.4 Hence, calculate which term in the sequence is the first to be greater than 35 (3)

[8]

Activity 2- March 2015

QUESTION 2

Beads are placed next to each other to form the arrangements as shown below:



Arrangement

Arrangement

Arrangement

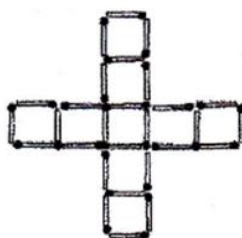
- 2.1 Write down how many beads will there be in the next two arrangements (2)
- 2.2 Determine an expression for the n -th term of the arrangement (2)
- 2.3 Determine how many beads will there be in the 150th arrangement (2)
- 2.4 Determine the arrangement in which there will be 448 beads (2) **[8]**

QUESTION 3

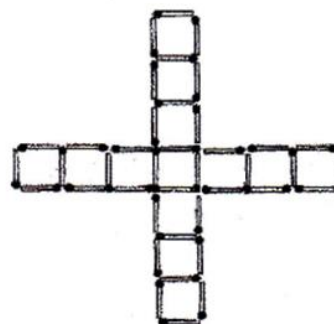
3.1 Daniel builds patterns that consist of squares as shown in the pictures below:



Picture 1



Picture 2



Picture 3

Assuming that the pattern is continued, answer the following questions:

3.1.1 How many squares will Picture 5 have? (2)

3.1.2 Determine an algebraic statement to generalise the relationship between the number of squares and the picture number (i.e. find the n^{th} term). (2)

3.1.3 Hence calculate the number of squares in the 31st picture. (2)

3.2 Write down the next TWO terms in each of the following number patterns:

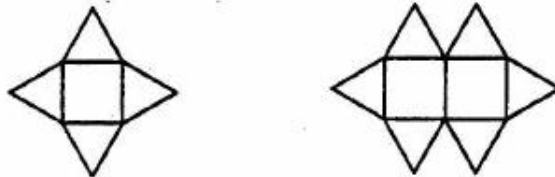
3.2.1 12; 9; 6; 3; ... (2)

3.2.2 4; 4; 8; 12; ... (2)

[10]

QUESTION 4

4.1 Study the following patterns, made with matches:



- 4.1.1 Draw the pattern that would have come next in the sequence. (2)
- 4.1.2 Use these diagrams to work out a general formula for the pattern, in terms of the number of matches.
Explain clearly how you arrived at your pattern. Write the answer in the form of $T_n = \dots\dots\dots$. (3)
- 4.1.3 Work out how many matches would be used to make the tenth pattern. (2)

References

1. Classroom Mathematics