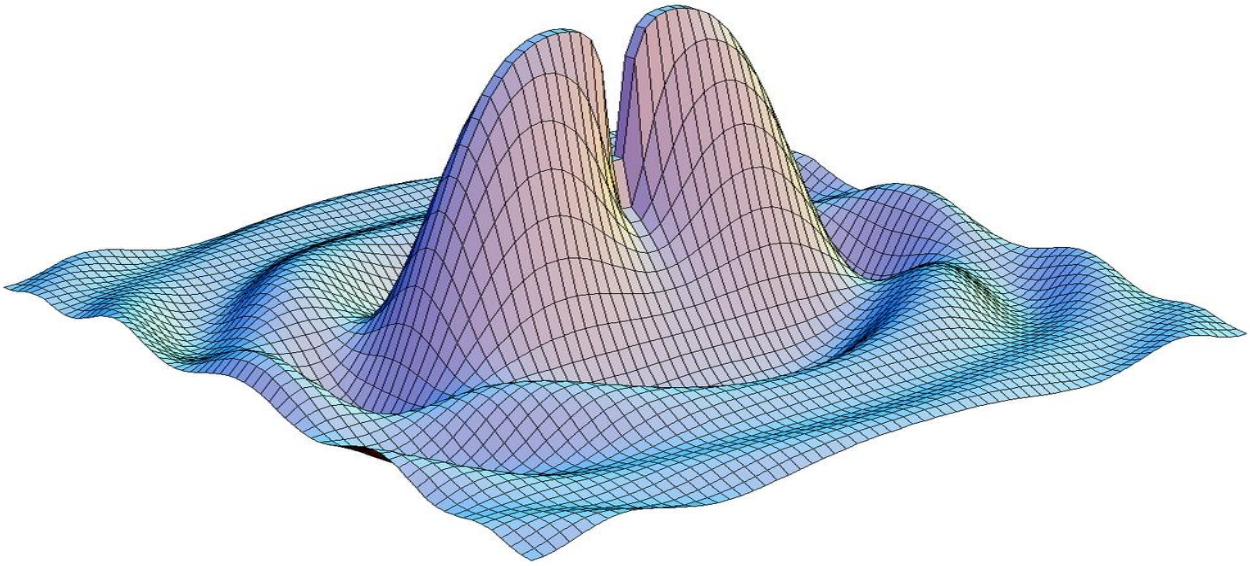


DEFFERENTIAL CALCULUS

MATHEMATICS GRADE 12

REVISION PACK (2019)

BY AYANDA DLADLA/ 074 994 7970

PAST PAPERS

FEB/MARCH 18

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=4x^2$. (5)

8.2 Determine:

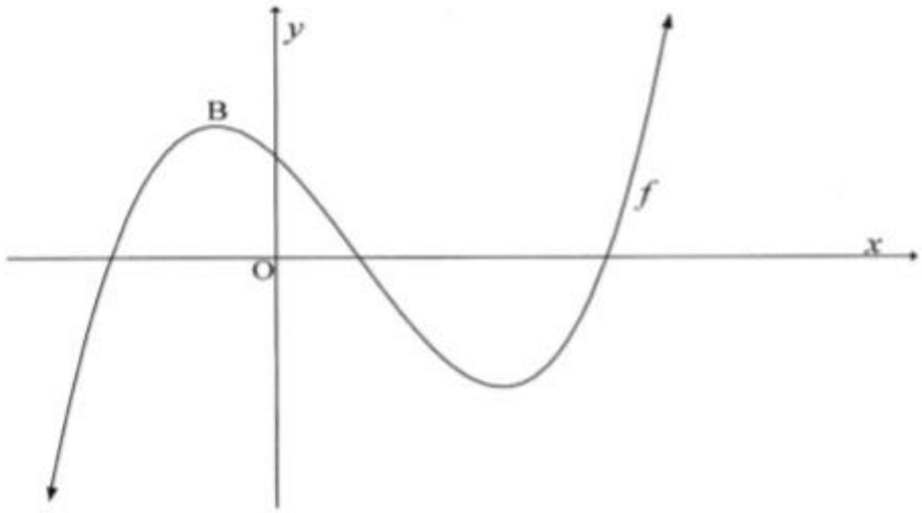
8.2.1 $D_x\left[\dfrac{x^2-2x-3}{x+1}\right]$ (3)

8.2.2 $f''(x)$ if $f(x)=\sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1 Calculate the values of b, c and d. (4)

9.2 Calculate the x-coordinate of B, the maximum turning point of f. (4)

9.3 Determine an equation for the tangent to the graph of f at $x=-1$. (4)

9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

QUESTION 10

Given: $f(x) = -3x^3 + x$.

Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$. **[6]**

NOV 18

QUESTION 8

8.1 Determine $f^{/}(x)$ from first principles if it is given $f(x) = x^2 - 5$. (5)

8.2 Determine $\frac{dy}{dx}$ if:

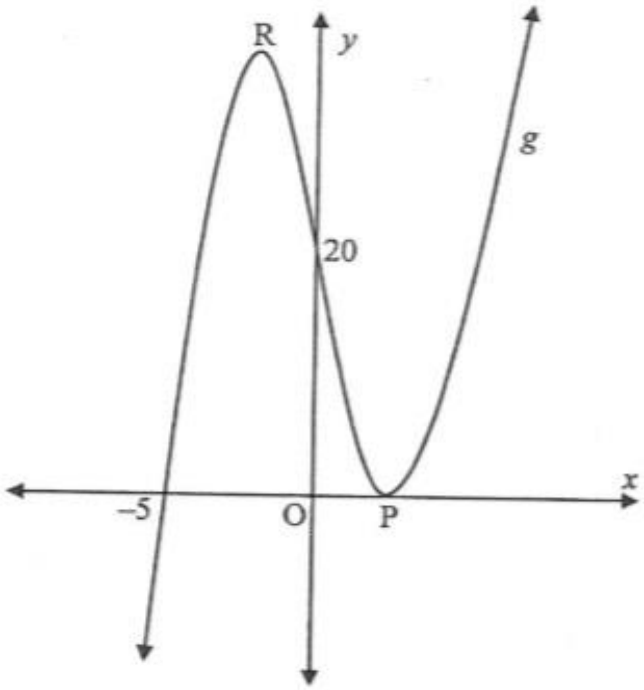
8.2.1 $y = 3x^3 + 6x^2 + x - 4$ (3)

8.2.2 $yx - y = 2x^2 - 2x \quad ; \quad x \neq 1$ (4)

[12]

QUESTION 9

9.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
The graph of g intersects the x-axis at $(-5 ; 0)$ and at P, and the y-axis at $(0 ; 20)$.
P and R are turning points of g.



9.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)

9.1.2 Calculate the coordinates of P and R. (5)

9.1.3 Is the graph concave up or concave down at $(0 ; 20)$? Show ALL your calculations. (3)

9.2 If g is a cubic function with:

- $g(3) = g'(3) = 0$
- $g(0) = 27$
- $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,

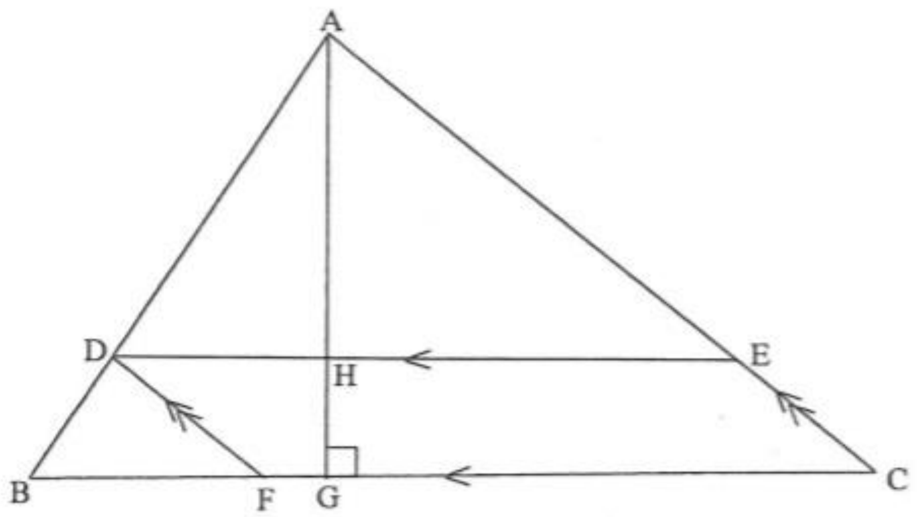
draw a sketch graph of g indicating ALL relevant points. (3)

[15]

QUESTION 10

In ΔABC:

- D is a point on AB, E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- $BF : FC = 2 : 3$.
- The perpendicular height AG is drawn intersecting DE at H.
- $AG = t$ units.
- $BC = (5 - t)$ units.



10.1 Write down $AH : HG$. (1)

10.2 Calculate t if the area of the parallelogram is a maximum.
(**NOTE:** Area of a parallelogram = base × $\perp$ height) (5)

[6]

SEPT 2018 MCED

QUESTION 8

8.1 Given: $f(x) = 3 - x^2$

8.1.1 Determine $f'(x)$ from first principles. (5)

8.1.2 Determine the gradient of the tangent to f where $x = -1$. (2)

8.2 Differentiate with respect to x:

8.2.1 $f(x) = -3x^3 - 2\sqrt{x}$ (3)

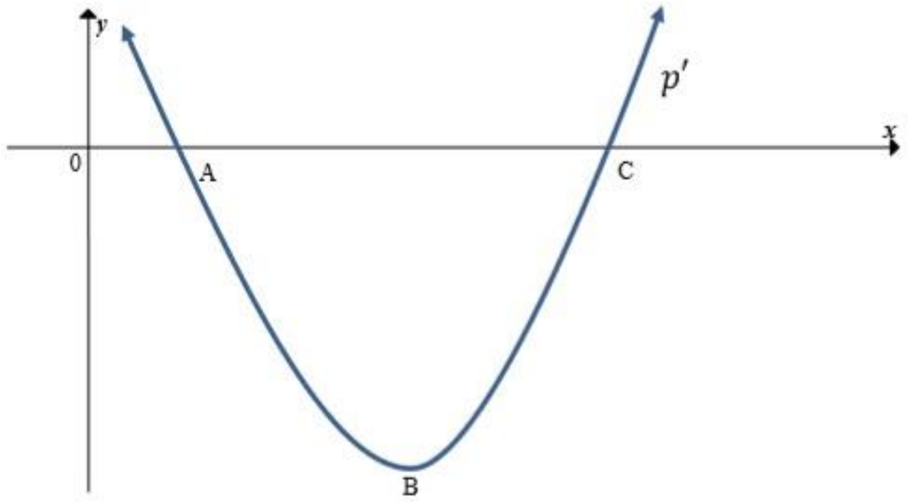
8.2.2 $xy = \left(x - \frac{1}{x^2}\right)\left(x + \frac{1}{x^2}\right)$ (4)

8.3 Given the functions $h(x) = \frac{k}{x}$ and $g(x) = 3x + 6$

8.3.1 Determine the equation of $h'(x)$ in terms of k. (2)

8.3.2 Calculate the value of k if g is a tangent to h. (5)

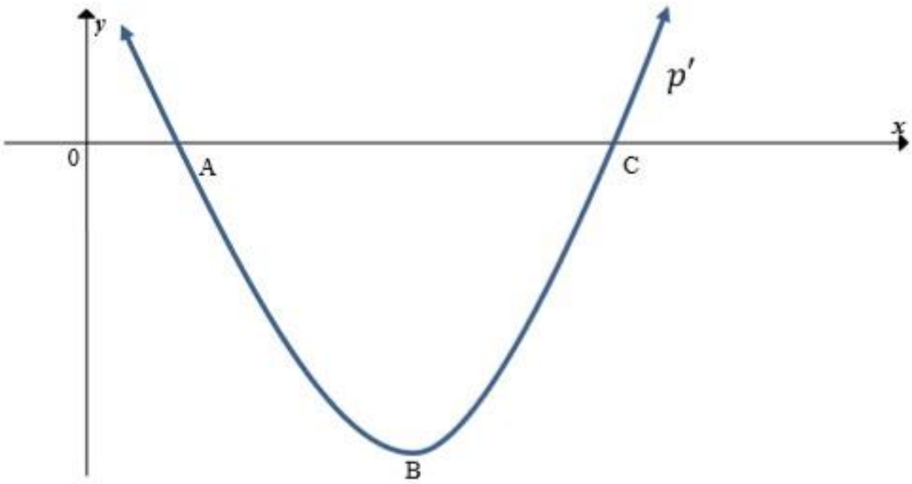
8.4 The sketch below shows the graph of $p'(x)$ where $p(x) = x^3 + bx^2 + 24x + c$. A(2;0) is an x-intercept of both $p(x)$ and $p'(x)$. C is the other x-intercept of $p'(x)$.



8.4.1	Show that the numeric value of $b = -9$. Clearly show all your calculations.	(3)
8.4.2	Calculate the coordinates of C.	(3)
8.4.3	For which value(s) of x will $p(x)$ be increasing?	(3)
8.4.4	Calculate the value(s) of x for which the graph of p is concave up.	(2)
8.4.5	Sketch a possible graph of $p(x)$. Clearly indicate the x-coordinates of the turning points and the point of inflection.	(4)

[36]

8.4 The sketch below shows the graph of $p'(x)$ where $p(x) = x^3 + bx^2 + 24x + c$. A(2;0) is an x-intercept of both $p(x)$ and $p'(x)$. C is the other x-intercept of $p'(x)$.



8.4.1 Show that the numeric value of $b = -9$. Clearly show all your calculations. (3)

8.4.2 Calculate the coordinates of C. (3)

8.4.3 For which value(s) of x will $p(x)$ be increasing? (3)

8.4.4 Calculate the value(s) of x for which the graph of p is concave up. (2)

8.4.5 Sketch a possible graph of $p(x)$. Clearly indicate the x-coordinates of the turning points and the point of inflection. (4)

[36]

SEPT 18 NW

QUESTION 8

8.1 If $f(x) = 2x^2 - 5x + 3$, determine $f'(x)$ from first principles. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3 + 1}{x^3}$ (5)

[10]

QUESTION 9

9.1 Given: $f(x) = -2x^3 + 5x^2 + 4x - 3$

9.1.1 Calculate the coordinates of the x-intercepts of f if $f(3) = 0$. Show all calculations. (4)

9.1.2 Calculate the x-values of the stationary points of f. (4)

9.1.3 For which values of x is f concave up? (2)

9.2 The function g, defined by $g(x) = ax^3 + bx^2 + cx + d$ has the following properties:

- $g(-2) = g(4) = 0$
- The graph of $g'(x)$ is concave up.
- The graph of $g'(x)$ has x-intercepts at $x = 0$ and $x = 4$ and a turning point at $x = 2$.

9.2.1 Use this information to draw a neat sketch graph of g without actually solving for a, b, c and d. Clearly show all x-intercepts, x-values of the turning points and x-value of the point of inflection on your sketch. (4)

9.2.2 For which values of x will $g(x) \cdot g''(x) > 0$? (3)

[17]

QUESTION 10

A shopkeeper finds that the number of people visiting his shop at any moment during the 10 hours that the shop is open, is represented by:

$$N(t) = t^3 - 12t^2 + 36t + 8\ ,$$

where $N(t)$ is the number of people in the shop, t hours after the shop opened.

10.1 How many people are in the shop when the shop opens? (1)

10.2 At what stage is the number of people in the shop increasing? (5)

10.3 At which stage is it the best time for the shopkeeper to take a break and leave his assistant alone in the shop? (1)

[7]

FEB/MARCH 17

QUESTION 7

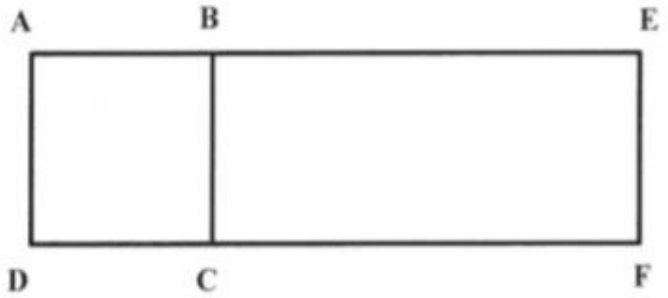
7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$. (5)

7.2 Determine the derivative of: $g(x) = 5x^2 - \frac{2x}{x^3}$ (3)

7.3 Given: $h(x) = ax^2$, $x > 0$.
Determine the value of a if it is given that $h^{-1}(8) = h'(4)$. (6)
[14]

QUESTION 9

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. **[7]**

NOV 17

QUESTION 7

7.1 Given: $f(x) = 2x^2 - x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$ (4)

[12]

QUESTION 8

Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

8.1 Show that f has a point of inflection at $x = 2$. (5)

8.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y = -f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2) + 3$. (2)

8.4.2 Claire claims that $f'(2) = 1$.

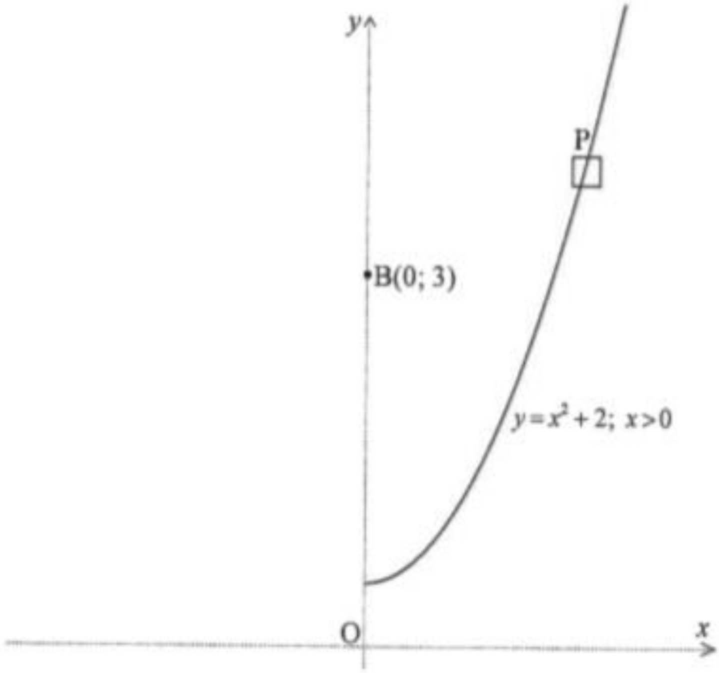
Do you agree with Claire? Justify your answer. (2)

[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

FEB/MARCH 16

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)

8.2 Determine the derivative of:

8.2.1 $y = 3x^2 + 10x$ (2)

8.2.2 $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)

8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

8.3.1 Prove that $(x-2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

8.3.3 Determine the x-coordinates of the turning points of f. (4)

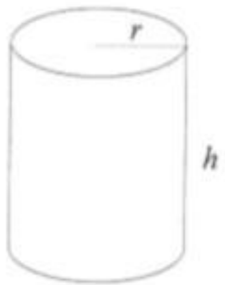
8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

8.3.5 Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer. (6)

[27]

QUESTION 9

A soft drink can has a volume of 340 cm^3, a height of h cm and a radius of r cm.



9.1 Express h in terms of r. (2)

9.2 Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$. (2)

9.3 Determine the radius of the can that will ensure that the surface area is a minimum. (4)

[8]

NOV 16

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2$ (5)

8.2 John determines $g'(a)$, the derivative of a certain function g at $x = a$, and arrives at the answer: $\lim_{h \to 0} \frac{\sqrt{4+h}-2}{h}$
Write down the equation of g and the value of a. (2)

8.3 Determine $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x^3}$ (4)

8.4 $g(x) = -8x + 20$ is a tangent to $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$. Calculate the values of a and b. (5)
[16]

QUESTION 9

For a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

9.1 Calculate the x-coordinates of the stationary points of f. (3)

9.2 For which values of x is f concave down? (3)

9.3 Determine the values of x for which f is strictly increasing. (2)

9.4 If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f. (5)
[13]

QUESTION 10

The number of molecules of a certain drug in the bloodstream t hours after it has been taken is represented by the equation $M(t) = -t^3 + 3t^2 + 72t$, $0 < t < 10$.

10.1 Determine the number of molecules of the drug in the bloodstream 3 hours after the drug was taken. (2)

10.2 Determine the rate at which the number of molecules of the drug in the bloodstream is changing at exactly 2 hours after the drug was taken. (3)

10.3 How many hours after taking the drug will the rate at which the number of molecules of the drug in the bloodstream is changing, be a maximum? (3)
[8]

FEB/MARCH 15

QUESTION 8

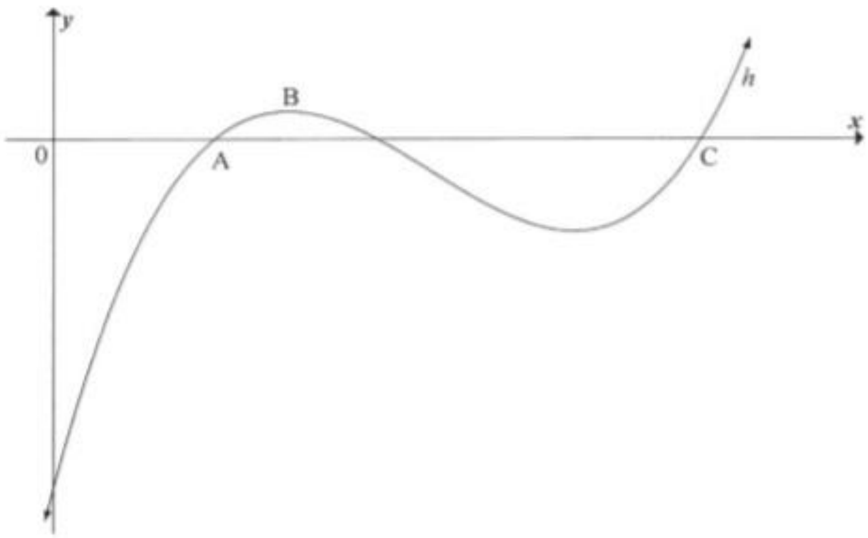
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

8.3.3 Calculate the coordinates of C. (4)

8.3.4 The graph of h is concave down for $x < k$. Calculate the value of k. (3)
[22]

QUESTION 9

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)

$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

9.1 If the volume of a cylinder is 6 cm^3, write h in terms of r. (1)

9.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$ (4)

9.3 Determine the value of r so that the least amount of paint will be used. (4)

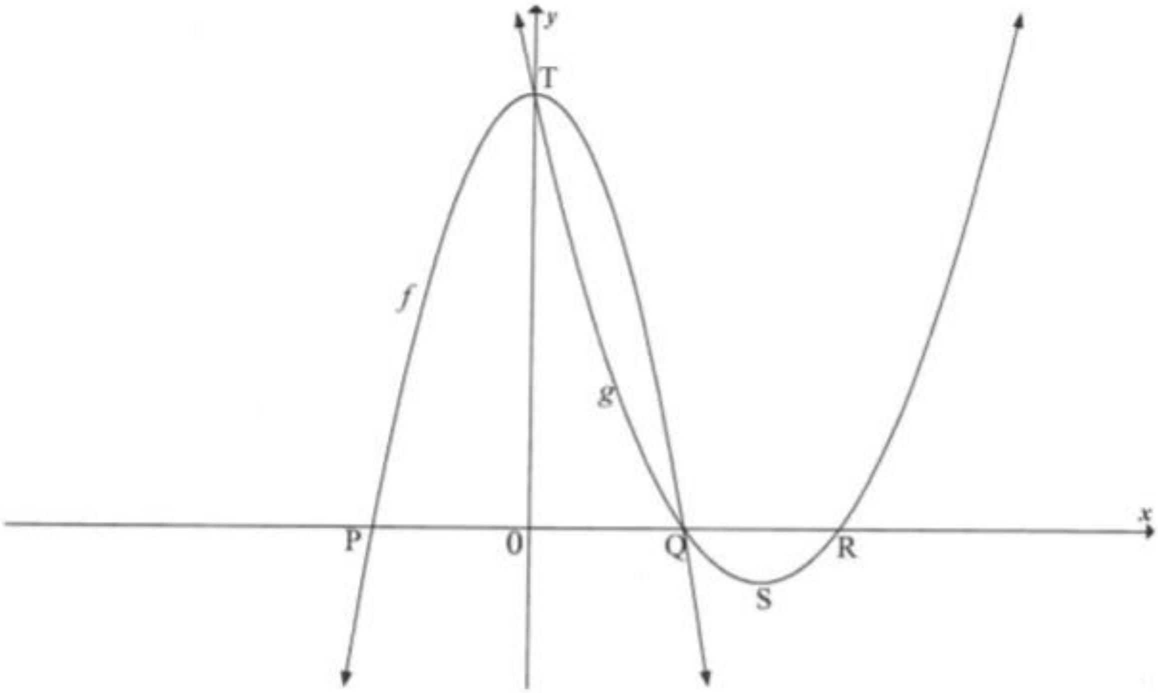
[9]

NOV 15

QUESTION 6

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

6.2 The function defined as $y = \dfrac{a}{x+p} + q$ has the following properties:

- The domain is $x \in R,\ x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R,\ x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

[12]

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$ (3)

[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q$(2\,;\,10)$ are the turning points of h. The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

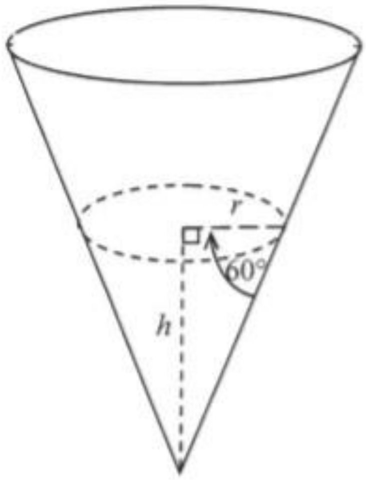
9.4 Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

[17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is $60°$, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\qquad$ $V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\qquad$ $V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

FEB/MARCH 14

QUESTION 10

10.1 Given: $f(x) = -\dfrac{2}{x}$

10.1.1 Determine $f'(x)$ from first principles. (5)

10.1.2 For which value(s) of x will $f'(x) > 0$? Justify your answer. (2)

10.2 Evaluate $\dfrac{dy}{dx}$ if $y = \dfrac{1}{4}x^2 - 2x$. (2)

10.3 Given: $y = 4\left(\sqrt[3]{x^2}\right)$ and $x = w^{-3}$

Determine $\dfrac{dy}{dw}$. (4)

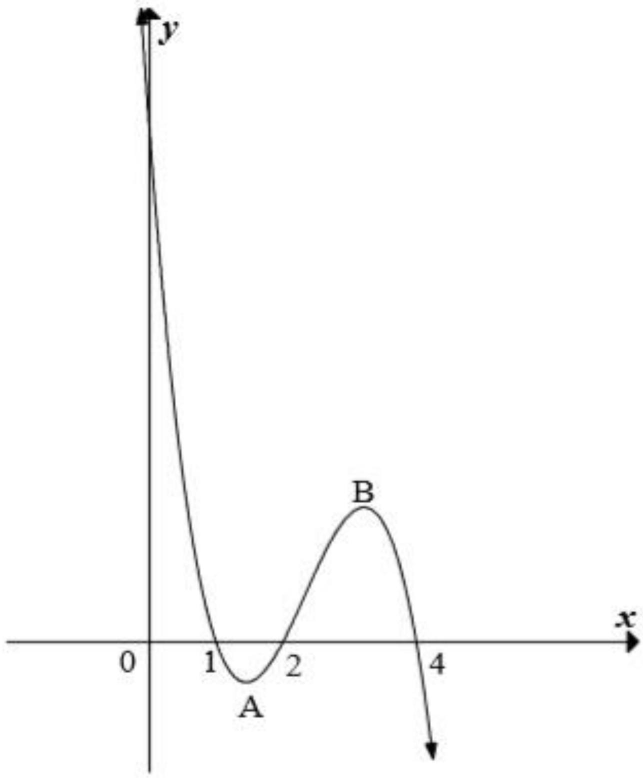
10.4 Given: $f(x) = ax^3 + bx^2 + cx + d$

Draw a possible sketch of $y = f'(x)$ if a, b and c are all NEGATIVE real numbers. (4)

[17]

QUESTION 11

The graph of $f(x) = -x^3 + ax^2 + bx + c$ is sketched below. The x-intercepts are indicated.



11.1 Calculate the values of a, b and c. (4)

11.2 Calculate the x-coordinates of A and B, the turning points of f. (5)

11.3 For which values of x will $f'(x) < 0$? (3)

[12]

QUESTION 12

A small business currently sells 40 watches per year. Each of the watches is sold at R144. For each yearly price increase of R4 per watch, there is a drop in sales of one watch per year.

12.1 How many watches are sold x years from now? (1)

12.2 Determine the annual income from the sale of watches in terms of x. (3)

12.3 In what year and at what price should the watches be sold in order for the business to obtain a maximum income from the sale of watches? (4)

[8]

NOV 14

QUESTION 8

8.1 Determine $f^{/}(x)$ from first principles if $f(x)=x^3$. (5)

8.2 Determine the derivative of: $f(x)=2x^2+\frac{1}{2}x^4-3$ (2)

8.3 If $y=\left(x^6-1\right)^2$, prove that $\frac{dy}{dx}=12x^5\sqrt{y}$, if $x>1$. (3)

8.4 Given: $f(x)=2x^3-2x^2+4x-1$. Determine the interval on which f is concave up. (4)

[14]

QUESTION 9

Given: $f(x)=(x+2)(x^2-6x+9)$
$=x^3-4x^2-3x+18$

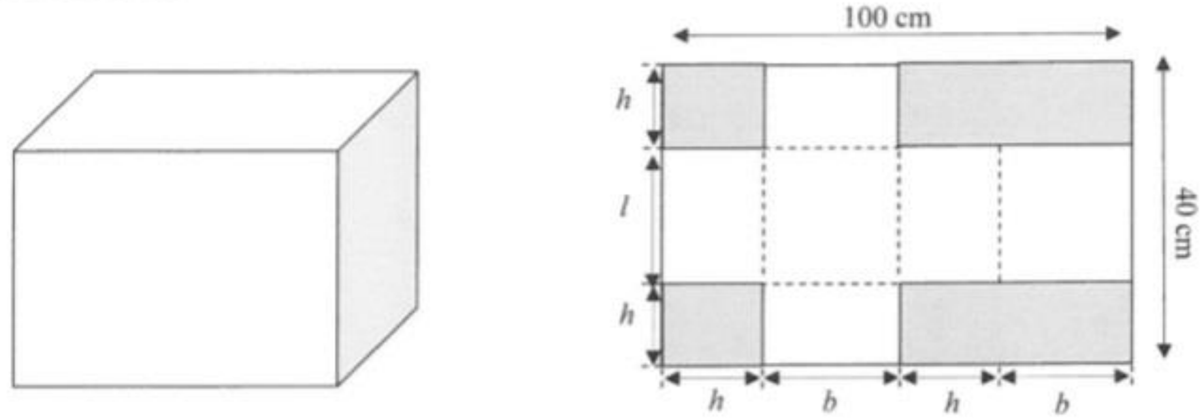
9.1 Calculate the coordinates of the turning points of the graph of f. (6)

9.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

9.3 For which value(s) of x will $x.f^{/}(x)<0$? (3)

[13]

QUESTION 10



A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

10.1 Express the length l in terms of the height h. (1)

10.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$ (3)

10.3 For which value of h will the volume of the box be a maximum? (5)
[9]

EXEMPL 14

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = 2x^{-4} - \frac{x}{5}$. (2)
[7]

QUESTION 9

Given: $f(x) = x^3 - 4x^2 - 11x + 30$.

9.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (4)

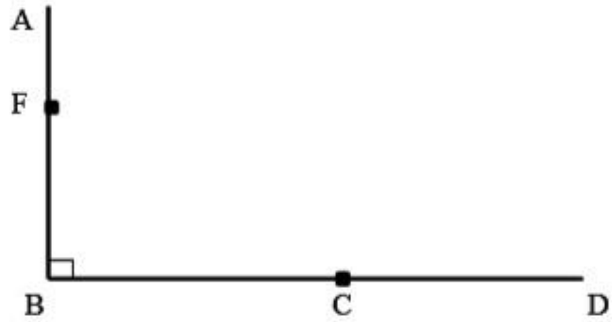
9.3 Calculate the coordinates of the stationary points of f. (5)

9.4 Sketch the curve of f in your ANSWER BOOK. Show all intercepts with the axes and turning points clearly. (3)

9.5 For which value(s) of x will $f'(x) < 0$? (2)
[15]

QUESTION 10

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



10.1	Determine the distance between F and C in terms of t.	(4)
10.2	After how long will the two cyclists be closest to each other?	(4)
10.3	What will the distance between the cyclists be at the time determined in QUESTION 10.2?	(2)
		[10]

FEB/MARCH 13

QUESTION 9

9.1	Use the definition of the derivative (first principles) to determine $f'(x)$ if $f(x) = 2x^3$	(5)
9.2	Determine $\frac{dy}{dx}$ if $y = \frac{2\sqrt{x}+1}{x^2}$	(4)
9.3	Calculate the values of a and b if $f(x) = ax^2 + bx + 5$ has a tangent at $x = -1$ which is defined by the equation $y = -7x + 3$	(6)
		[15]

QUESTION 10

Given: $f(x) = -x^3 - x^2 + x + 10$

10.1 Write down the coordinates of the y-intercept of f. (1)

10.2 Show that $(2\ ;\ 0)$ is the only x-intercept of f. (4)

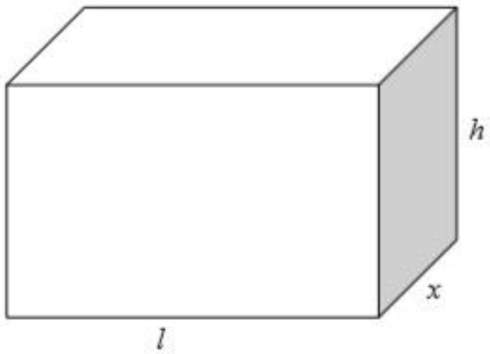
10.3 Calculate the coordinates of the turning points of f. (6)

10.4 Sketch the graph of f in your ANSWER BOOK. Show all intercepts with the axes and all turning points. (3)

[14]

QUESTION 11

A rectangular box is constructed in such a way that the length (l) of the base is three times as long as its width. The material used to construct the top and the bottom of the box costs R100 per square metre. The material used to construct the sides of the box costs R50 per square metre. The box must have a volume of 9 m^3. Let the width of the box be x metres.



11.1 Determine an expression for the height (h) of the box in terms of x. (3)

11.2 Show that the cost to construct the box can be expressed as $C = \frac{1200}{x} + 600x^2$. (3)

11.3 Calculate the width of the box (that is the value of x) if the cost is to be a minimum. (4)

[10]

NOV 13

QUESTION 8

8.1 Given: $f(x) = 3x^2 - 4$

8.1.1 Determine $f'(x)$ from first principles. (5)

8.1.2 A(x ; 23), where $x > 0$, and B(-2 ; y) are points on the graph of f. Calculate the numerical value of the average gradient of f between A and B. (5)

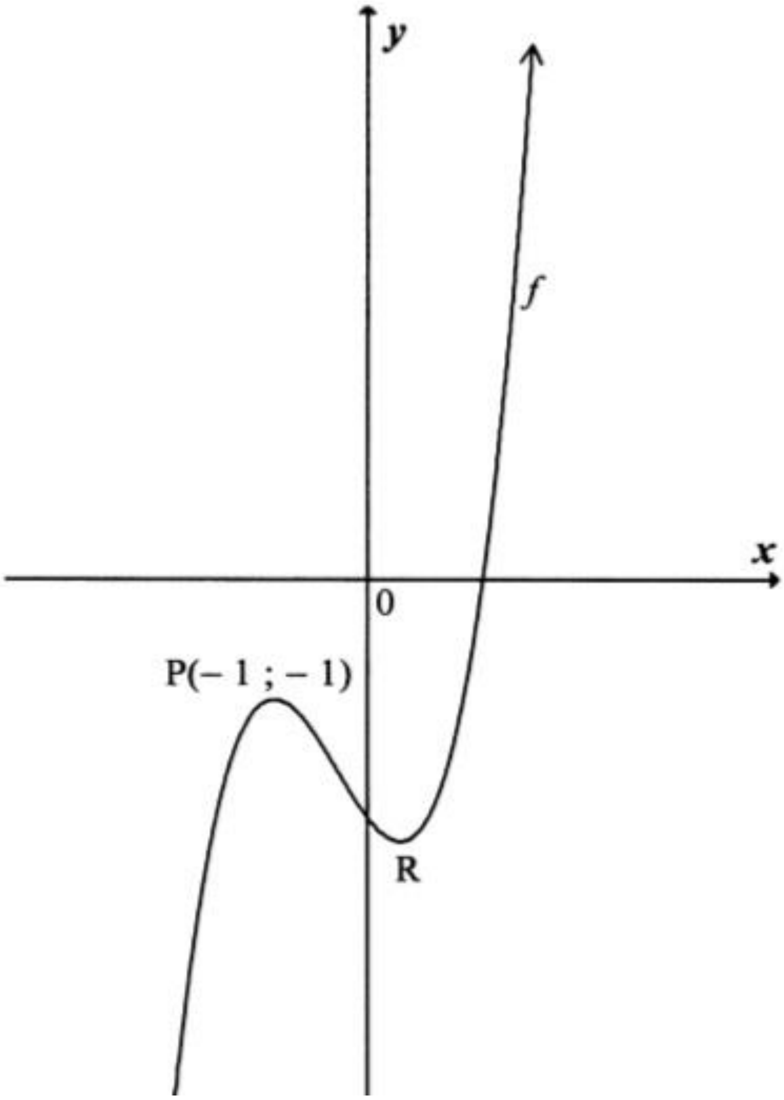
8.2 Differentiate $y = \frac{x+5}{\sqrt{x}}$ with respect to x. (3)

8.3 Determine the gradient of the tangent of the graph of $f(x) = -3x^3 - 4x + 5$ at $x = -1$. (4)

[17]

QUESTION 9

The function defined by $f(x)=x^3+ax^2+bx-2$ is sketched below.
P(– 1 ; – 1) and R are the turning points of f.



9.1 Show that $a=1$ and $b=-1$. (6)

9.2 Hence, or otherwise, determine the x-coordinate of R. (3)

9.3 Write down the coordinates of a turning point of h if h is defined by $h(x)=2f(x)-4$. (2)

[11]

QUESTION 10

An industrial process requires water to flow through its system as part of the cooling cycle. Water flows continuously through the system for a certain period of time.

The relationship between the time (t) from when the water starts flowing and the rate (r) at which the water is flowing through the system is given by the equation:

$$r = -0{,}2t^2 + 10t$$

where t is measured in seconds.

10.1 After how long will the water be flowing at the maximum rate? **(3)**

10.2 After how many seconds does the water stop flowing? **(3)**
[6]

FEB/MARCH 12

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 9 - x^2$. (5)

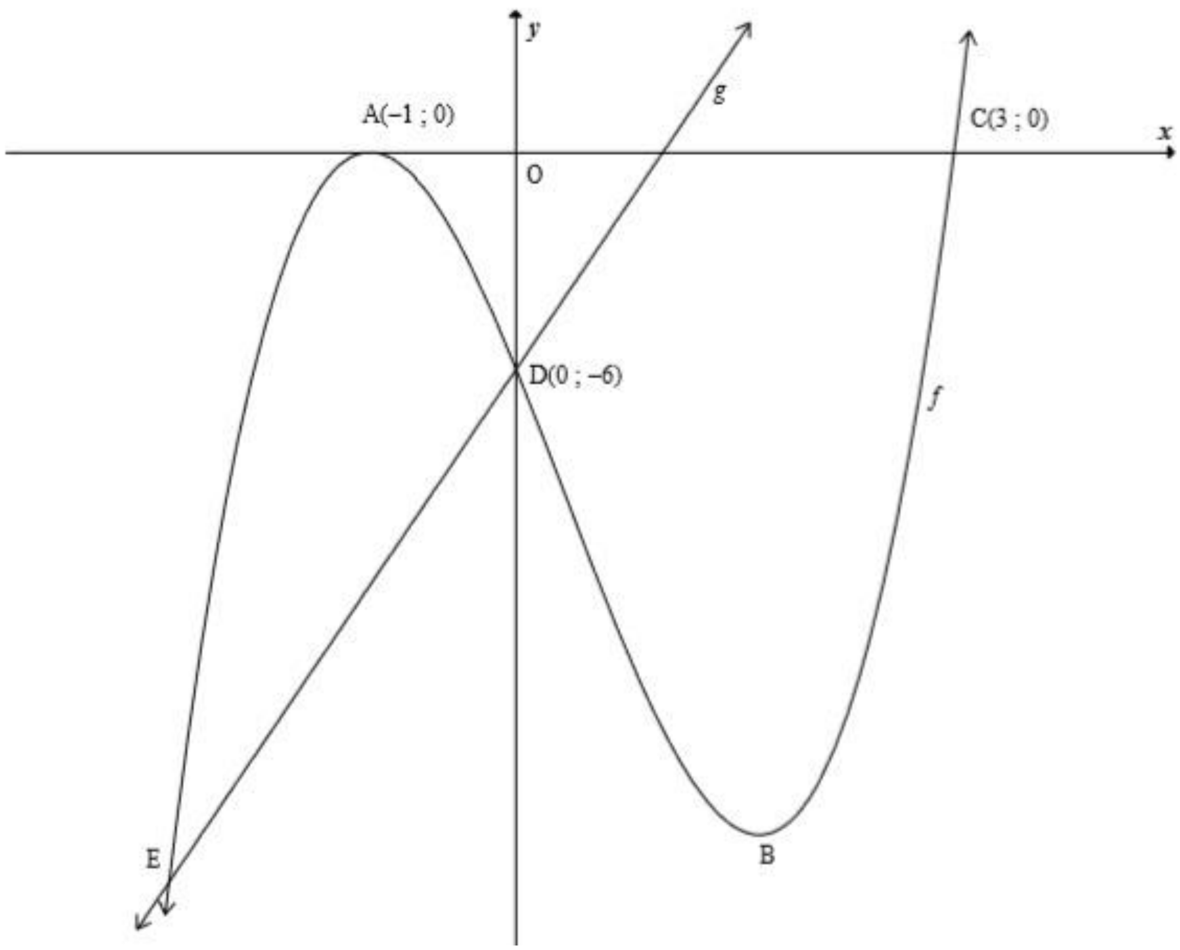
8.2 Evaluate:

8.2.1 $D_x[1 + 6\sqrt{x}]$ (2)

8.2.2 $\dfrac{dy}{dx}$ if $y = \dfrac{8 - 3x^6}{8x^5}$ (4)
[11]

QUESTION 9

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A$(-1\ ;\ 0)$ and C$(3\ ;\ 0)$ are the x-intercepts of f.
The graph of f has turning points at A and B.
D$(0\ ;\ -6)$ is the y-intercept of f.
E and D are points of intersection of the graphs of f and g.



9.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$. (5)

9.2 Calculate the coordinates of the turning point B. (5)

9.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$. Calculate x such that $h(x)$ is a maximum, where $x < 0$. (5)

[15]

QUESTION 10

The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

10.1 Show that $(1\,;-3)$ is the point of contact of the tangent to the graph. (1)

10.2 Hence or otherwise, calculate the values of p and q. (6)
[7]

QUESTION 11

A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$
- f decreases for $x \in \left[-\frac{1}{3}\,;2\right]$ only

Draw a possible sketch graph of f, clearly indicating the x-coordinates of the turning points and ALL the x-intercepts. **[4]**

NOV 12

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 5$. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y = x^{-4} + 2x^3 - \frac{x}{5}$. (3)

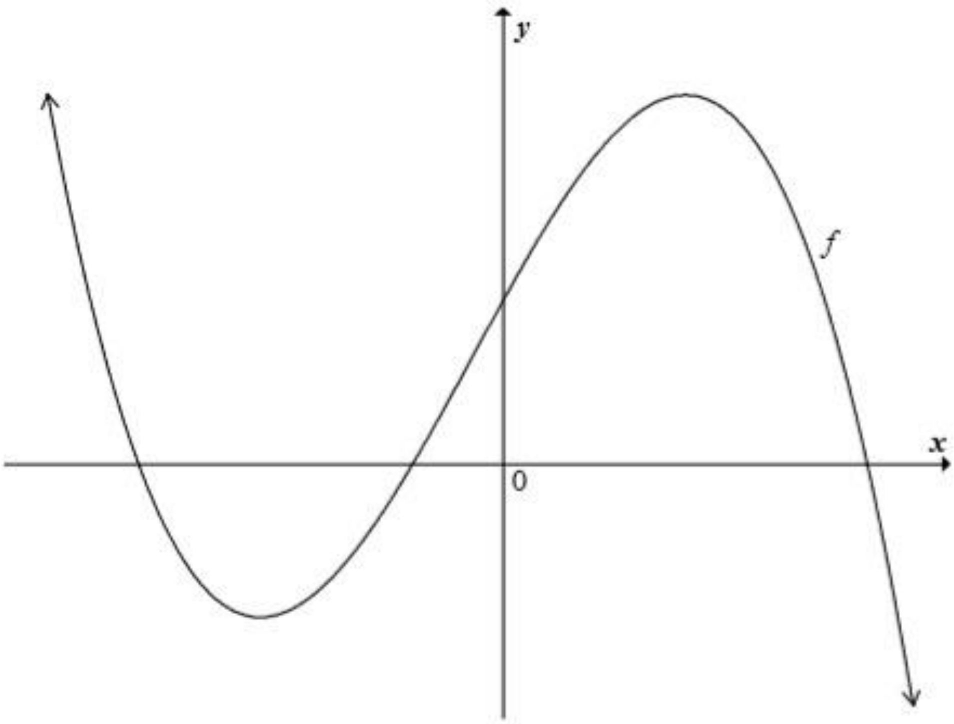
8.3 Given: $g(x) = \frac{x^2 + x - 2}{x - 1}$

8.3.1 Calculate $g'(x)$ for $x \neq 1$. (2)

8.3.2 Explain why it is not possible to determine $g'(1)$. (1)
[11]

QUESTION 9

9.1 The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.



9.1.1 Calculate the x-coordinates of the turning points of f. (4)

9.1.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

9.2 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

9.2.1 Determine the equation of the tangent to the graph of g at $x = -1$. (4)

9.2.2 For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

9.3 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations. (3)

[17]

QUESTION 10

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

10.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.) (3)

10.2 Determine the rate at which the velocity of the particle is changing at t seconds. (1)

10.3 After how many seconds will the particle be closest to the fixed point? (2)

[6]

FEB/MARCH 11

QUESTION 9

9.1 Use the definition to differentiate $f(x) = 1 - 3x^2$. (Use first principles.) (4)

9.2 Calculate $D_x\left[4 - \frac{4}{x^3} - \frac{1}{x^4}\right]$. (3)

9.3 Determine $\frac{dy}{dx}$ if $y = \left(1 + \sqrt{x}\right)^2$. (3)

[10]

QUESTION 10

Given: $g(x) = (x-6)(x-3)(x+2)$

10.1 Calculate the y-intercept of g. (1)

10.2 Write down the x-intercepts of g. (2)

10.3 Determine the turning points of g. (6)

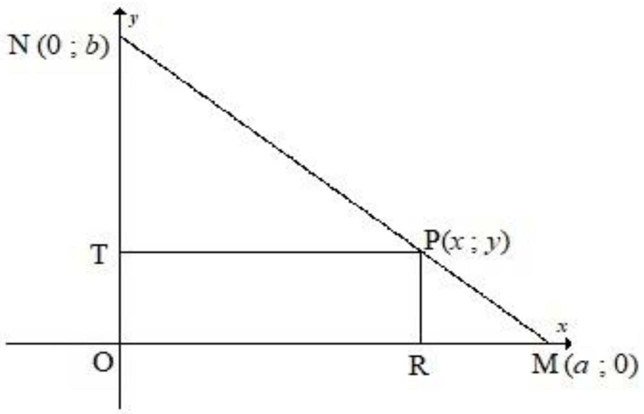
10.4 Sketch the graph of g on DIAGRAM SHEET 2. (4)

10.5 For which values of x is $g(x).g^{/}(x) < 0$? (3)

[16]

QUESTION 11

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = a, ON = b and P(x ; y) be any point on MN.



11.1 Determine an equation of MN in terms of a and b. (2)

11.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

[8]

NOV 11

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$. (5)

8.2 Evaluate:

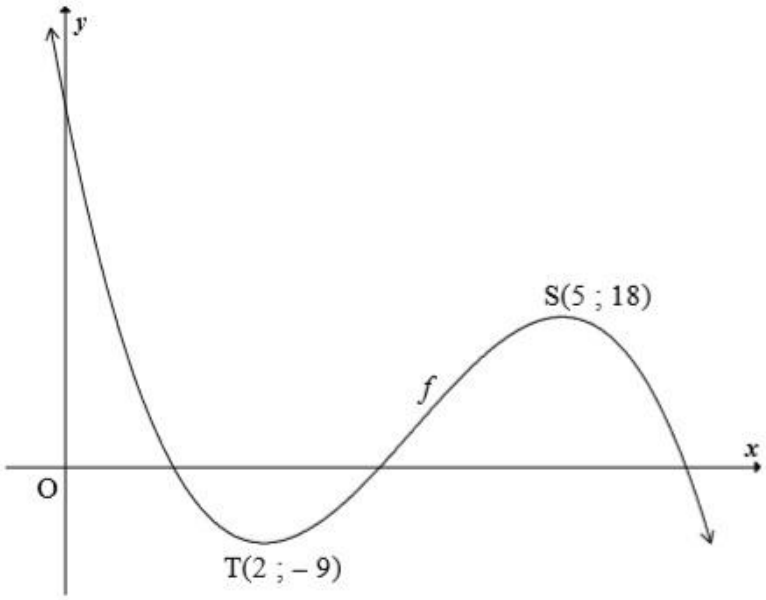
8.2.1 $\frac{dy}{dx}$ if $y = \frac{3}{2x} - \frac{x^2}{2}$ (3)

8.2.2 $f'(1)$ if $f(x) = (7x+1)^2$ (4)

[12]

QUESTION 9

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.
The turning points of the graph of f are $T(2\,;-9)$ and $S(5\,;18)$.



9.1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)

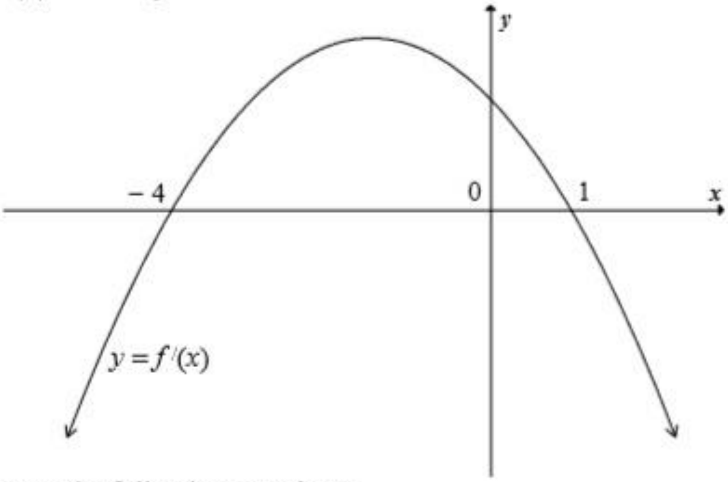
9.2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)

9.3 Determine the x-value at which the graph of f has a point of inflection. (2)

[14]

QUESTION 10

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



Use the graph to answer the following questions:

10.1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)

10.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)

[4]

QUESTION 11

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula $V(t) = 100 - 4t$.

11.1 What is the initial volume of the water in the tank? (1)

11.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)

11.3 Determine the value of k (that is, the rate at which water flows out of the tank). (2)

[6]

FEB/MARCH 10

QUESTION 10

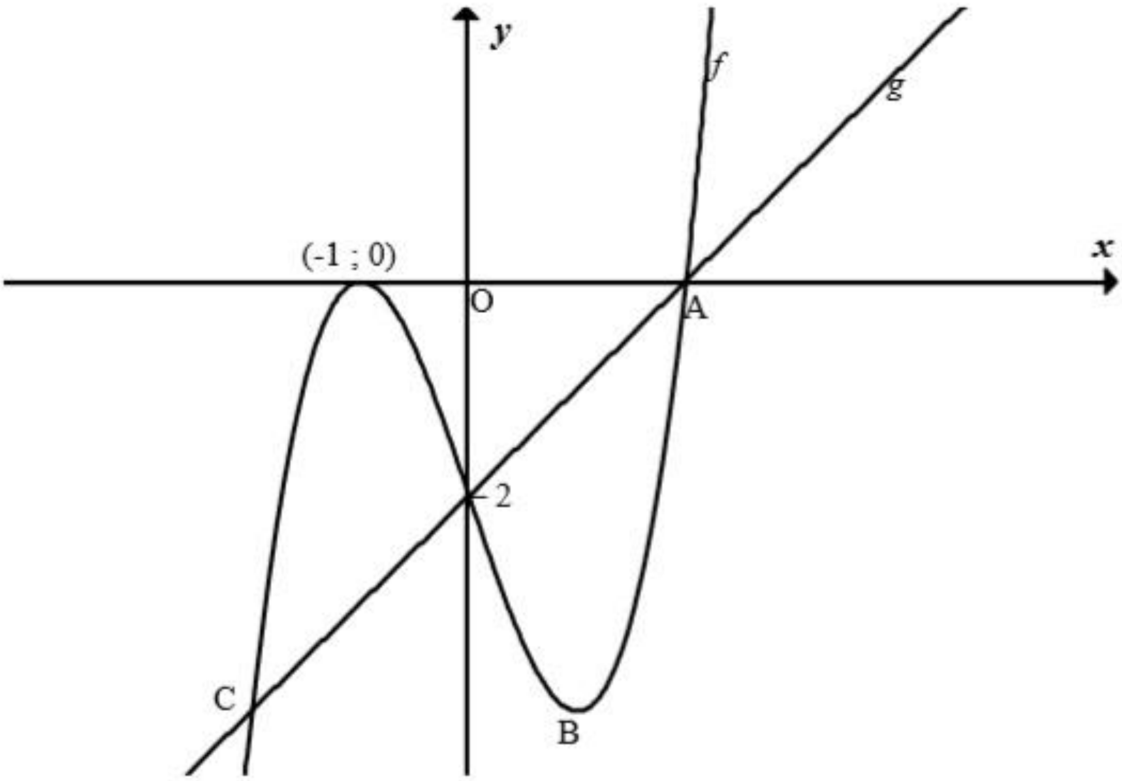
10.1 Differentiate f from first principles: $f(x) = \frac{1}{x}$ (4)

10.2 Use the rules of differentiation to determine $\frac{dy}{dx}$ if $y = (2 - 5x)^2$ (3)

[7]

QUESTION 11

The graph below represents the functions f and g with $f(x) = ax^3 - cx - 2$ and $g(x) = x - 2$.
A and $(-1\ ;\ 0)$ are the x-intercepts of f. The graphs of f and g intersect at A and C.



11.1	Determine the coordinates of A.	(1)
11.2	Show by calculation that $a = 1$ and $c = -3$.	(4)
11.3	Determine the coordinates of B, a turning point of f.	(3)
11.4	Show that the line BC is parallel to the x-axis.	(7)
11.5	Find the x-coordinate of the point of inflection of f.	(2)
11.6	Write down the values of k for which $f(x) = k$ will have only ONE root.	(3)
11.7	Write down the values of x for which $f'(x) < 0$.	(2)
		[22]

QUESTION 12

A wire, 4 metres long, is cut into two pieces. One is bent into the shape of a square and the other into the shape of a circle.

12.1 If the length of wire used to make the circle is x metres, write in terms of x the length of the sides of the square in metres. (1)

12.2 Show that the sum of the areas of the circle and the square is given by $f(x) = \left(\frac{1}{16} + \frac{1}{4\pi}\right)x^2 - \frac{x}{2} + 1$ square metres. (4)

12.3 How should the wire be cut so that the sum of the areas of the circle and the square is a minimum? (3)

[8]

NOV 10

QUESTION 8

8.1 Differentiate $g(x) = x^2 - 5$ from first principles. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{x^6}{2} + 4\sqrt{x}$. (3)

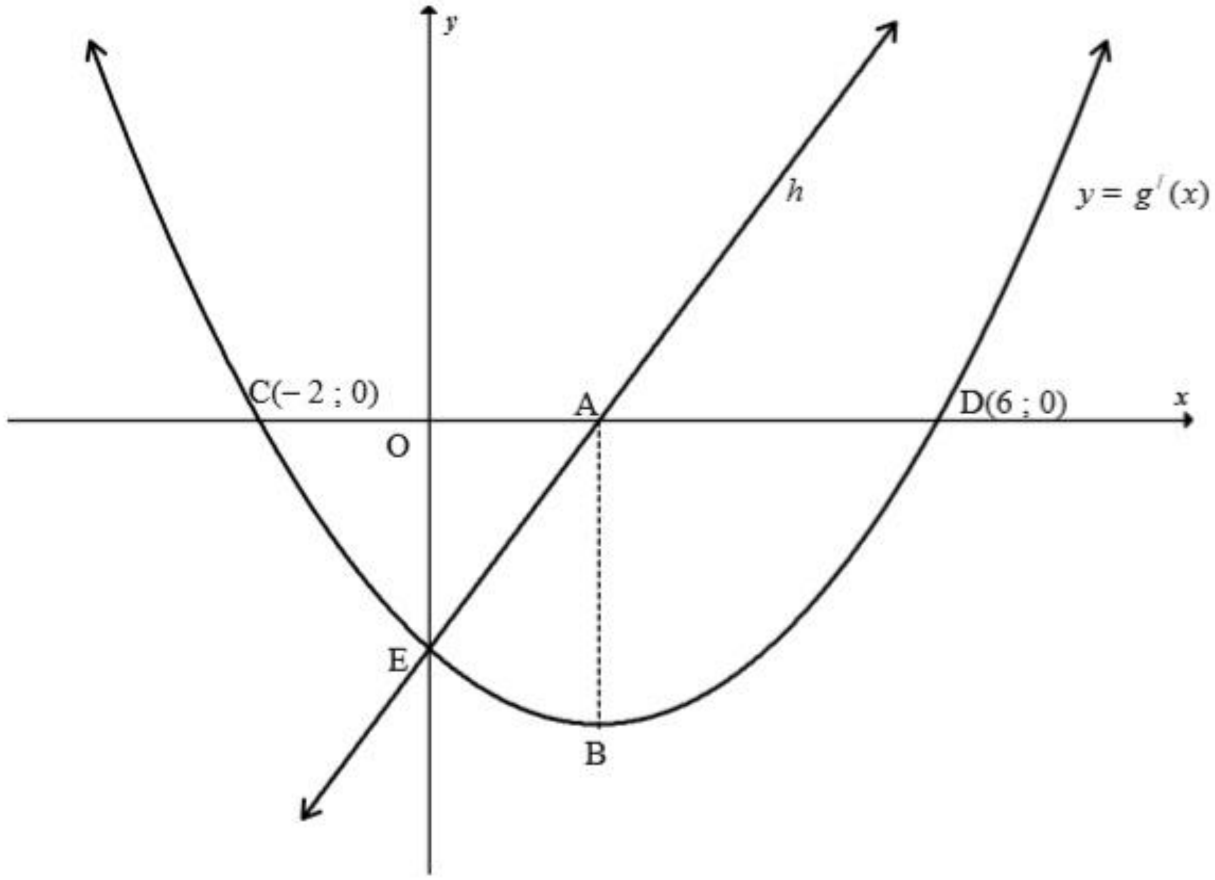
8.3 A function $g(x) = ax^2 + \frac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96. Calculate the values of a and b. (6)

[14]

QUESTION 9

The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g.

The graphs of h and g' have a common y-intercept at E.
C$(-2 ; 0)$ and D$(6 ; 0)$ are the x-intercepts of the graph of g'.
A is the x-intercept of h and B is the turning point of g'.
AB $\parallel$ y-axis.

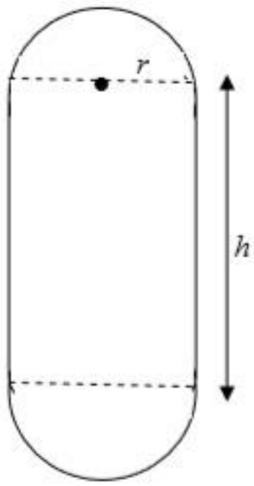


9.1	Write down the coordinates of E.	(1)
9.2	Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$.	(4)
9.3	Write down the x-coordinates of the turning points of g.	(2)
9.4	Write down the x-coordinate of the point of inflection of the graph of g.	(2)
9.5	Explain why g has a local maximum at $x = -2$.	(3)
		[12]

QUESTION 10

A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive.

The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere = $4\pi r^2$
Curved surface area of a cylinder = $2\pi rh$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Volume of a cylinder = $\pi r^2 h$

NOV 09

QUESTION 9

9.1 Differentiate f from first principles if it is given that $f(x) = -5x^2 + 2$ (5)

9.2 Use the rules of differentiation to determine the following:

$D_x[(x-2)(x+3)]$ (3)

9.3 The depth h of petrol in a large tank, t days after the tank was refilled, is given by $h(t) = 12 - \frac{t}{4} - \frac{t^3}{6}$ metres for $0 \leq t \leq 4$.

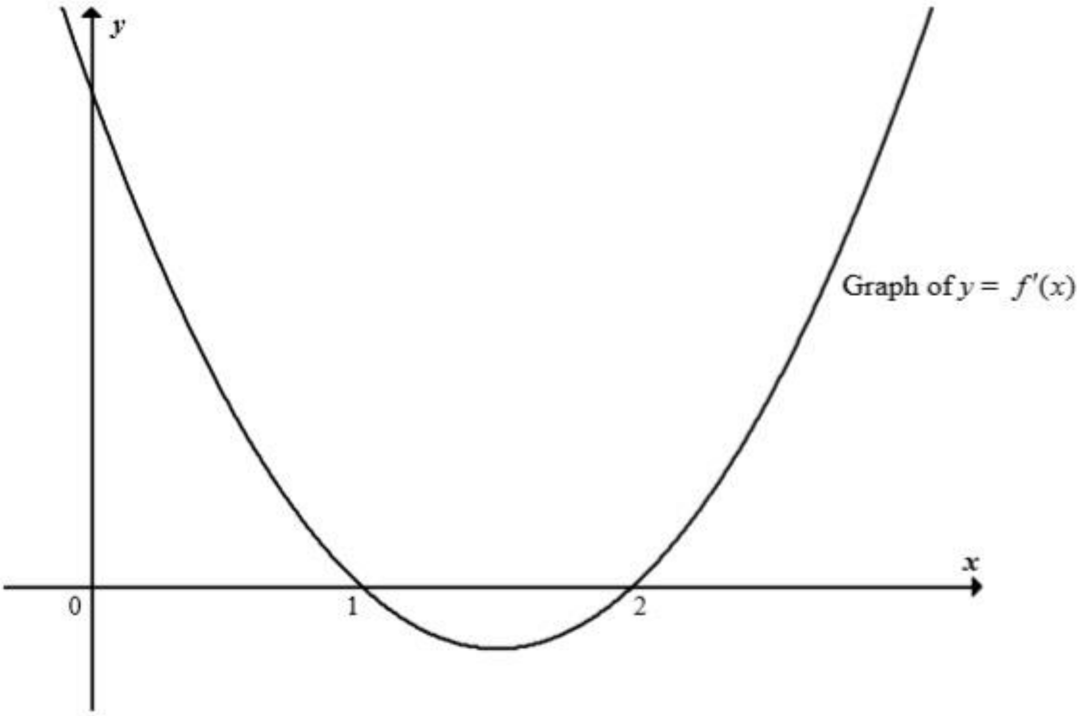
9.3.1 What is the depth after 3 days? (1)

9.3.2 What is the rate of decrease in the depth after 2 days? (Give your answer in the correct units.) (5)

[14]

QUESTION 10

In the sketch below, the graph $y = ax^2 + bx + c$ represents the derivative, f', of f where f is a cubic function.

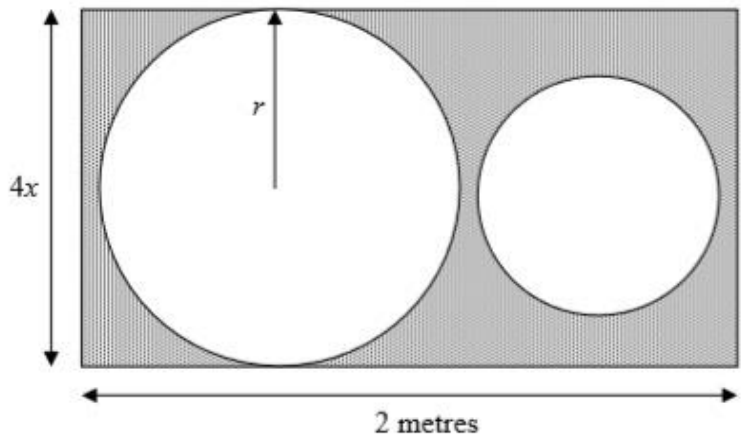


10.1	Write down the x-coordinates of the stationary points of f.	(2)
10.2	State whether each stationary point in QUESTION 10.1 is a local minimum or a local maximum. Substantiate your answer.	(4)
10.3	Determine the x-coordinate of the point of inflection of f.	(1)
10.4	Hence, or otherwise, draw a sketch graph of f.	(2)
		[9]

QUESTION 11

Devan wants to cut two circles out of a rectangular piece of cardboard of 2 metres long and $4x$ metres wide. The radius of the larger circle is half the width of the cardboard and the smaller circle has a radius that is $\frac{2}{3}$ the radius of the bigger circle.

$A = lb$	$A = \pi r^2$	$P = 2(l+b)$	$C = 2\pi r$



11.1 Show that the area of the shaded region is $A(x) = 8x - \frac{52\pi}{9}x^2$. (5)

11.2 Determine the value of x, such that the area of the shaded region is a maximum. (3)

11.3 Calculate the total area of the circles, if the area of the shaded region is to be a maximum. (2)

[10]

EXEMPL 08

QUESTION 9

9.1 Determine $f'(x)$ from first principles if $f(x) = \dfrac{1}{x}$ (5)

9.2 Determine the derivatives of:

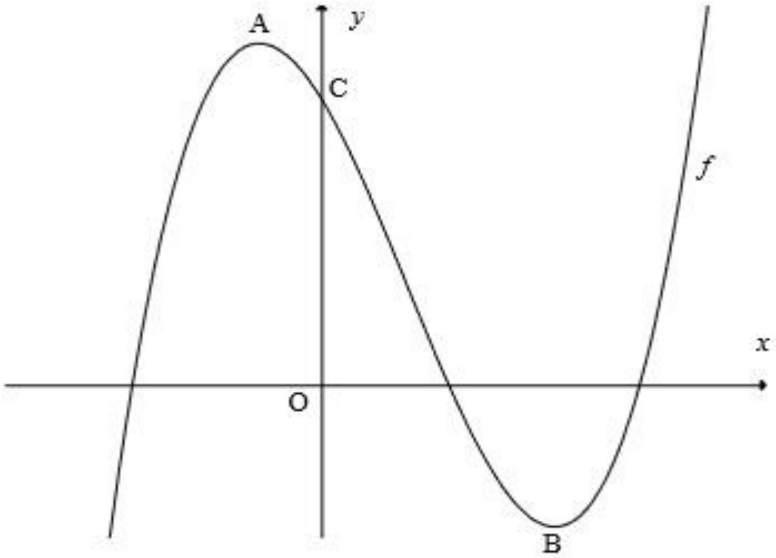
9.2.1 $f(x) = -5x^2 + 2x$ (2)

9.2.2 $y = \sqrt{x^3} + \dfrac{1}{3x^3}$ (4)

[11]

QUESTION 10

Sketched is the graph of $f(x) = x^3 - 4x^2 - 11x + 30$
A and B are the turning points of f.



10.1 Determine the coordinates of A and B. (5)

10.2 Determine the turning points of g if $g(x) = f(x-2)$ (2)

10.3 Determine the average rate of change of the function f from A to B. (3)

10.4 Determine the equation of the tangent to the graph of f at $x = 1$. (4)

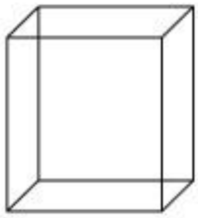
10.5 Determine the x-coordinate of the point at which the tangent in QUESTION 10.4 cuts the graph of f again. (4)

10.6 Determine the value(s) of k for which $x^3 - 4x^2 - 11x + 30 = k$ will have only one real root. (2)

10.7 Determine the point(s) of inflection of f. (6)

[26]

QUESTION 11



The volume of a certain rectangular box is given by the equation $f(x) = x^3 - 8x^2 + 5x + 50$

11.1 If the height of the box is $(5 - x)$ units, determine an algebraic expression for the area of the base of the box. (3)

11.2 Calculate the value of x for which the volume is a maximum. (6)
[9]

ADD EXEMPL 08

QUESTION 10

10.1 Determine $f'(x)$ by first principles if $f(x) = x^3$ (5)

10.2 Use differentiation rules to differentiate the following:

10.2.1 $y = \dfrac{2}{5\sqrt{x}} - \sqrt[3]{x}$ (4)

10.2.2 $y = \dfrac{x^4 - 3x^2 + 7}{x}$ (4)
[13]

QUESTION 11

Given: $f(x) = x^3 + x^2 - 5x + 3$

11.1 Calculate the x- and y-intercepts of f. (5)

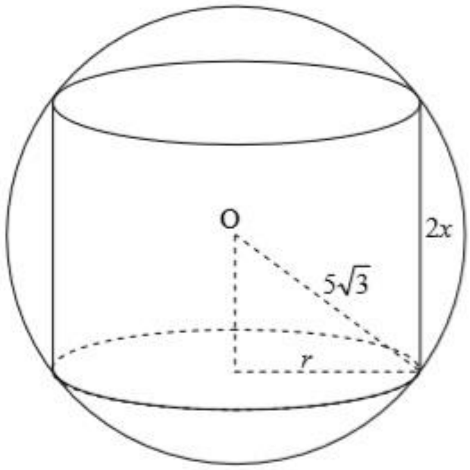
11.2 Determine the turning points of f. (5)

11.3 Determine the x-value of the point of inflection. (3)

11.4 Hence, sketch the graph of f on DIAGRAM SHEET 1. Show clearly ALL intercepts with the axes and any turning points. (3)
[16]

QUESTION 12

A cylinder with height $2x$ units is placed inside a sphere with radius $5\sqrt{3}$ units.
O is the centre of the sphere.



12.1 Show that the volume of the cylinder can be expressed as $V = 150\pi x - 2\pi x^3$. (3)

12.2 Calculate the height of the cylinder if it is of maximum volume. (4)

[7]