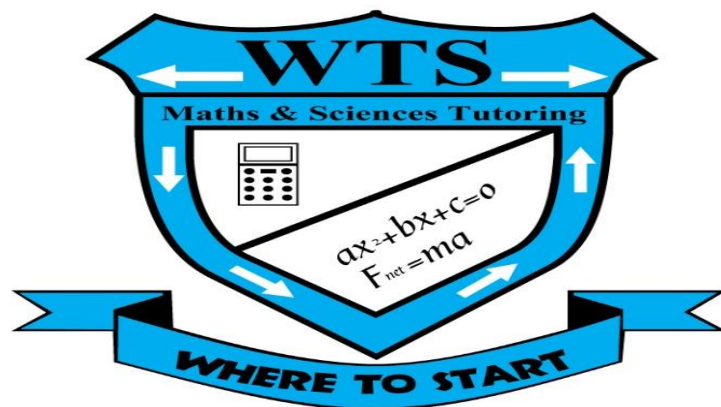


WTS TUTORING



FINANCIAL MATHS

GRADE : 10 TO 12

COMPILED BY : kwv"BABE'SWEMATHS/MASTERMATHS" SIBIYA

CELL NO. : 0826727928

EMAIL : kwvsibiya@gmail.com

FACEBOOK P. : WTS MATHS & SCEINCE TUTORING

DAY : ONE**FINANCIAL MATHEMATICS**

It deals with two types of interest rate:

- simple interest
- compound interest

GRADE 10**1. SIMPLE/LINEAR METHOD**

Shape: Indicate (A, P & n)

a) Appreciation

b) Depreciation

It is based only on the amount of money invested/borrowed and not on balanced basis.

For example, consider R200 invested for 4 years at 20% annum simple interest.

Formula:.....

If $A > P$

Appreciation

If $A < P$

Depreciation

A

future/accumulated/investment amount (NB-will/to etc)

P

Present/principal /invested amount (NB-is/was, cost)

n

term loan (NB-stated in years)

i

interest (rate =100i)

Equal instalment using simple interest rate:

$$x = \frac{A}{n \cdot m}$$

Note: $A = n \cdot m \cdot x$

The interest earned: $I = A - P$

The percentage of the interest earned: $I\% = \frac{I}{P} \cdot 100$

Example 1

Mr Kwv Sibiya invests R45 000 for 12 years at an interest rate of 13% per annum simple interest. Calculate:

- a) the accumulated amount of the investment in 12 years time.
- b) the simple interest received at the end of the 12th year.
- c) the simple interest received each year.

Kwv 1

Sbu invests R14000 for 9 years at simple interest rate of 8% per annum. Calculate;

- a) the future value of the investment
- b) the simple interest received at the end of the 9th year.
- c) the simple interest received each year.

Example 2

Ntando started to save money six years ago. The current value of her investment is R38 000. The interest rate for the investment was 7% per annum simple interest. How much did she invest six years ago?

Kwv 2

Nathi wants to invest a sum of money now so as to afford a new play station costing R5000 in three years time. She receives a simple interest rate of 6% per annum. Calculate this amount for Nathi.

Example 3

Mandla invested R6000 and it accumulated to R10 000 after 3 years. Find the interest rate if the investment earned simple interest. Round off your answer to one decimal place.

Kwv 3

Zethembe invests R3000 and it accumulates to R6000 over a period of 5 years. What simple interest rate would he need to secure?

Kwv 4

Calculate how long it would take for an investment of R9000 to double if the simple interest rate is 11% per annum

Kwv 5

An amount of R1000 is invested at 15%p.a. simple interest. A further R 1200 is invested at 10% p.a. simple interest. By when will both investments have the same accumulated values?

2. HIRE PURCHASE AGREEMENTS

A hire purchase agreement (HP) is a short-term loan. Furniture and household appliances, such as refrigerators, are generally bought on HP. Buyers sign an agreement with the seller to pay a specified amount per month. The interest paid on a hire purchase loan is simple interest and it is calculated on the full value of the loan over the repayment period. Normally a deposit is paid initially and the balance is paid over a short time period.

Take note of the following:

Loan = *cost* – *deposit*

Total amount paid off to the loan: $Tp = x.n + \textit{deposit}$

Example 1

Hlengiwe buys a tumble dryer for R4000. She takes out a hire purchase loan involving equal monthly payments over 3 years. The interest rate charged is 14% per annum simple interest. She also takes out an insurance premium of R12, 40 per month to cover the cost of damage or theft. Calculate:

- the actual amount paid for the tumble dryer.
- the interest paid.
- how much must be paid each month.

Kwv 1

Jimmy buys a furniture suite for R9 800. He takes out a hire-purchase loan involving equal monthly payments over five years. The interest rate charged is 15% per annum simple interest. He also takes out an insurance premium of R12 per month to cover the cost of damage or theft. Calculate:

- the actual amount paid for the furniture suite
- the interest paid
- how much must be paid each month.

Example 2

Nombulelo buys a computer costing R12 000. She pays a 20% deposit and then takes out a 24 month hire purchase loan on the balance. The interest rate charged on the loan is 12% per annum simple interest. Calculate her monthly payments and what she will actually pay for the computer.

3. COMPOUND INTEREST

Shape: Indicate (A, P & n)

d) Appreciation

b) Depreciation

Interest paid on the initial principal and previously earned interest.

If interest is calculated on the original sum plus interest already earned, then it is called compound interest. For example, consider R100 invested for 3 years at 10% per annum simple interest.

Formula:,,,

Example 1

Sakhile invests R1800 for 6 years at 15% p.a. compounded annually. Find the future value of his investment after 6 years and the percentage interest he receives.

Kwv 1

A home presently costs R2 850 000. Calculate the future value of the home in 12 years time taking into account an inflation rate of 14% p.a.

Example 2

Jabulile has just opened a small coffee shop and takes out a loan to provide the initial capital to start the business. She agrees to repay the loan 4 years later by means of a payment of R800 000. The bank charges her an interest rate of 18% per annum compounded annually. What was the amount of money she originally borrowed?

Kwv 2

You want to travel overseas in 5 years from now. Inflation is 12% p.a. The trip is expected to cost R90 000 then. How much must you invest now in order to afford the trip?

Example 3

R6800 is invested for 6 years and grows in value to R12 500. Find the interest rate if interest is compounded annually.

Kwv 3

If salaries double every 7 years in order to keep up with inflation, what rate of inflation does this imply?

4. INFLATION

Inflation is the steady compounded increase in prices over time throughout the economy. The effect of inflation is to erode the buying power of money over time.

Example 1

A motor vehicle currently costs R400 000. If the rate of inflation is 11% p.a. compounded annually, how much will this car cost in 7 years' time?

5. EXCHANGE RATE

There are different money systems in different countries. Currency is the term used to describe the particular money system of a country. Here are the currencies of a few countries.

Country	Currency used	Symbol for the currency
South Africa	Rand	R
United States of America	US dollar	\$
United Kingdom	British Pound	£
Several European countries	Euro	€

In every country, in order to purchase goods and services, you would need to use their currency.

Example 1

Mzo wants to buy the latest DJ equipment, which has been advertised in a US catalogue for \$ 1400. He wants to order and pay for the equipment online. The current rand/dollar exchange rate is R11, 43 to the US dollar. Calculate the cost in rands of the DJ equipment.

Example 2

Thembeke is on a trip to London to visit her uncle. The current rand/pound exchange rate is R13, 50 to the British pound. She has R30 000 to spend in England. How many pounds does she have to spend in England?

Example 3

The foreign exchange table below indicates the average exchange rate of the South African rand to other currencies.

FOREIGN EXCHANGE TABLE

COUNTRY	CURRENCY	AVERAGE EXCHANGE RATE OF THE RAND
United Kingdom (UK)	Pound (£)	11,85
United States of America (USA)	Dollar (\$)	6,30

A company in South Africa exports steel tables to the UK and USA. The total cost to manufacture a table in South Africa is R275, 00 per table.

- How much, in dollars (\$), will a company in the USA pay for a table?
- How much, in pounds (£), will a company in the UK pay for a table?
- If the UK could manufacture the same tables at a price of £20 each; will they still buy from South Africa? Show ALL your calculations to substantiate your answer.
- As a result of increasing costs in manufacturing the tables, South Africa wishes to increase the prices by 10%. How much will the USA now pay for a single table in dollars?

Kwv 1

A DVD player costs \$256 in California. What would it cost in South Africa if the rand/dollar exchange rate is R10,45 to the US dollar?

Kwv 2

You want to buy a book online costing £35 in London. How much will it cost you in rands? The rand/pound exchange rate is R13 to the pound.

Kwv 3

You want to buy a box of Japanese sweets costing 50 yen (¥). If the rand/ yen exchange rate is one rand to 16,78 yen, calculate the cost of the sweets in rands.

Kwv 4

Sam is visiting a friend in New York for a week. He has R2 500 to spend and will exchange the money for US dollars. How many dollars will he have to spend if the rand/dollar exchange rate is R10,38 to the US dollar?

GRADE 11**1. DEPRECIATION****Key words:**

Book value-is the value of equipment at a given time after depreciation has taken place.

Scrap value-is the book value of the equipment at the end of its useful life.

1.1 SIMPLE INTEREST

It depreciates based on the straight line method.

1.2 COMPOUND INTEREST

It depreciates based on the reducing balanced basis.

Example 1

Sthabiso wants to sell his car in five years time. The rate of depreciation is 14% per annum and the car's current value is R60 000. Calculate the book value of the car in 5 years time, if depreciation is based on:

- a) the straight –line method?
- b) the reducing-balance method?

Example 2

Calculate the original price of a computer if its depreciated value after 7 years is R900 and the rate of depreciation was 12% annum calculated using:

- a) the straight-line method?
- b) the reducing-balance method?

Example 3

A laptop cost R9 000 and, after 4 years, has a scrap value of R2000. Find the annual depreciation rate if it is calculated using:

- a) the straight-line method.
- b) the reducing-balance method.

Kwv 1

May buys a motor car for R160 000. The car depreciates at a rate of 11% p.a. what is the car worth after 6 years if depreciates

Kwv 2

Find the future value of R4 000 invested for 5 years at

- (a) 14% p.a. simple interest.
- (b) 14% p.a. compound interest.

Kwv 3

Find the present value of an amount which accumulated to R13 000 in 6 years if the interest was

- (a) 12% p.a. simple interest.
- (b) 12% p.a. compound interest.

Kwv 4

R70 000 is invested at 9% p.a. simple interest for 3 years. Thereafter, the total amount is reinvested in a different financial institution at 8% p.a. compound interest for 2 more years.

What is the future value of the investment after the five-year period?

Kwv 5

Use the graph to answer the following questions:

- (a) Determine the value of the car after 7 years:
- (b) Determine the value of the car after 9 years:
- (c) How long did the car take to depreciate to R60 000?
- (d) How long did the car take to depreciate to R40 000?
- (e) What will the value of the car be after 10 years?

Kwv 6

A car is purchased for R250 000. The car depreciates at a rate of 12% per annum. What is the car worth after 5 years if depreciation is calculated using:

- (a) the straight-line method.
- (b) the reducing-balance method.

Kwv 7

Calculate the original price of a laptop computer if its depreciated value after 7 years is R3 200 and the rate of depreciation was 12% per annum calculated

using:

- (a) the straight-line method?
- (b) the reducing-balance method?

Kwv 8

A photocopying machine costs R140 000 and has a scrap value of R19 000 after 10 years.

Find the annual rate of depreciation if it is calculated using:

(a) the straight-line method.

(b) the reducing-balance method.

Kwv 9

A motor vehicle currently has a book value of R56 000. The rate of depreciation was 14% per annum using the reducing balance method. Calculate the original price of the motor vehicle, if it was bought 5 years ago.

2. COMPOUNDING PERIODS

- Compounding periods (m)

Annually	:-----	Semi-annually	:-----
Monthly	:-----	After every three months	:-----
Half yearly	:-----	Quarterly	:-----
Weekly	:-----	After every six months	:-----
Daily	:-----		

NB: In any calculation that we do, we must work with the effective interest rate.

Effective interest rate:

- The rate where the stated period and the compounding periods are the same

Nominal interest rate:

- The rate where the stated period and the compounding period are not the same

Take note:

- Effect of compounding period:
Term loan and interest must be converted
- The units are very important:

The interest earned: $I = A - P$

The percentage of the interest earned: $I\% = \frac{I}{P} \cdot 100$

Example 1

Calculate the future value of an investment of R25 000 after 3 years at an interest rate of 18% per annum compounded:

- a) annually
- b) half-yearly
- c) quarterly
- d) monthly

Example 2

What amount must be invested for 2 years at an interest rate of 10% per annum compounded half-yearly in order to receive R10 000?

Example 3

Suppose that R4000 is invested for 6 years at an interest rate of 16% per annum compounded monthly. Calculate the value of the investment at the end of the 6 year period.

2. CALCULATING THE INTEREST RATE**If money:**

Double, or triple the initial:-----OR -----(APP OR DEP)

Quarter or became certain % of its original amount:-----OR -----(APP OR DEP)

Example 1

After 4 years of reducing balance depreciation, an asset has a quarter of its original value. The original value was R86 000. Calculate the depreciation interest rate, as a percentage. (Correct your answer to 1 decimal place.)

Example 2

- a) A computer is purchased for R16 000. It depreciates at 15% per annum.
- b) Determine the book value of the computer after 3 years if depreciation is calculated according to the straight-line method.
- c) Find the rate, according to the reducing-balance method, that would yield the same book value as in QUESTION b) after 3 years.

Kwv 1

Calculate the future value of an investment of R20 000 after 6 years, if the interest rate is 15% per annum compounded:

- (a) annually
- (b) half yearly
- (c) quarterly
- (d) monthly
- (e) daily (excluding leap years)

Kwv 2

How much more money would you have over a period of 15 years on a savings of R60 000 if the interest rate of 12% per annum was compounded monthly, as compared to an interest rate of 12% compounded annually?

Kwv 3

R30 000 is invested for 4 years at an interest rate of 16% per annum compounded quarterly. Thereafter, the accumulated amount is reinvested for a further 5 years at an interest rate of 15% per annum compounded semi-annually (half yearly). Calculate the value of the investment at the end of the 9 year period.

Kwv 4

Determine which of the following two savings options is a better investment over a period of one year, if the interest is calculated at:

- A. 14% per annum compounded monthly
- B. 16% per annum compounded quarterly

Kwv 5

An amount of money was invested 6 years ago and it is now, after 6 years, worth R1 200 000. The interest rate for the savings period was 18% per compounded monthly. What was the amount that was originally invested 6 years ago?

3. THE TIME LINE

The value of P or A must be multiplied by an increasing or decreasing factor.

Calculating A : exponents must be positive (left to right)

Calculating P : exponents must be negative (right to left)

Example 1

Suppose that R4000 is invested for 6 years. The interest rate for the first 4 years is 16% per annum compounded quarterly. For the remaining two years, the interest rate changes to 18% per annum compounded monthly. Calculate the value of the investment at the end of the 6 year period.

Example 2

Suppose that R10709, 72783 is received if a certain amount of money was invested 6 years ago. The interest rate for the first 4 years was 16% per annum compounded quarterly. For the remaining 2 years, the interest rate changed to 18% per annum compounded monthly. What was the original amount invested?

Example 3

Philisiwe invests R16000 for 4 years at 10% per annum simple interest. Thereafter, she invests the accumulated amount for another 2 years at 12% per annum compounded monthly. Calculate how much money she will have saved at the end of the 6 year period.

Example 4

Nontobeko deposits R30 000 into a savings account. The interest rate for the first 3 years is 7% per annum compounded half-yearly. Thereafter, the interest rate changes to 8% per annum compounded quarterly. Calculate the value of the investment at the end of the 10 year.

Example 5

Bongumusa wants to have saved R4 000 000 in 8 years time. How much must he invest now if the interest rate for the first 6 years will be 6% per annum compounded monthly and 8% per annum compounded quarterly for the remaining 2 years.

Kwv 1

R5000 is deposited into a savings account. The interest rate for the first four years is 7% per annum compounded quarterly. Thereafter, the interest rate changes to 8% per annum compounded semi-annually. Calculate the value of the investment at the end of the tenth year.

Kwv 2

Mark invests R2000 for a period of seven years. During the first four years, the interest rate is 18% p.a. compounded monthly. Thereafter, interest changes to 24% p.a. compounded semi-annually. Calculate the future value of the investment after seven years.

Kwv 3

Simphiwe deposits R3000 into a savings account paying 13% per annum compounded monthly. After five years, the interest rate increases by 1%.

Three years later, the interest rate decreases by 2%. Calculate the value of her investment after ten years.

Kwv 4

Mvelo invests R6000 into an account for a period of 12 years. The interest rate for the first seven years is 8% per annum compounded monthly. For the next five years, the interest rate changes to 10% per annum compounded half-yearly.

(a) Convert the nominal rates to annual effective rates.

(b) Use the effective rates and calculate the future value of the savings at the end of the 12-year period.

4. Additional (deposit) and withdrawal of amount

For deposit : positive sign must be used

For withdrawal : negative sign must be used

Note : Treat each amount separately and grow it up to the last period.

Example 1

Skhumbuzo deposited R1000 into a bank. Three years later, he deposited a further R2000 into the bank. The interest rate for the 5 years savings period was 18% per annum compounded annually. Calculate the future value of his investment at the end of the savings period.

Example 2

Sihle deposits a gift of R20 000 into a savings account in order to save up for an overseas trip in 6 years time. The interest rate for the savings period is 8% per annum compounded monthly.

a) how much money will he have saved at the end of the six year period?

b) suppose that at the end of the second year, he withdraws R4 000 from the account. How much money will he have then saved at the of the six year period?

c) suppose that at the end of the third year, he adds R3 000 to the savings. How much money will he have then saved at the end of the six year period?

Example 3

Zinhle deposits R5 500 into a savings account. Three years later, R4 000 is added to the savings. The interest rate for the first 3 years is 14% per annum compounded semi-annually. Thereafter, the interest rate changes to 12% per annum compounded monthly. Calculate the future value of the savings at the end of the seventh year.

Example 4

Hloniphile deposits Rx into a unit trust savings account. Two years later, she deposits a further R2x into the account. Three years after this, she deposits R3x into the account. The interest rate for this 5 year period is 18% per annum compounded monthly. She receives R60 000 at the end of the year period. Calculate the value of x

Example 5

Zama takes out a loan to finance the purchasing of a new DVD player. He repays the loan means of a payment of R5 000 4 years after the granting of the loan. Two years later, he repays a final amount of R6 000. The interest rate during the first 4 years of the loan is 16% per annum compounded quarterly. For the remaining two years, the interest rate changes to 14% per annum compounded half-yearly. How much money did Zama originally borrow?

Kwv 1

R3 500 is deposited into a savings account. Three years later, R4 000 is added to the savings. The interest rate for the two years is 8% per annum compounded monthly. Thereafter, the interest rate changes to 10% per annum compounded semi-annually. Calculate the value of the savings at the end of the sixth year.

Kwv 2

Thabo deposits R5 000 into a savings account. Three years later, he deposits a further R4 000 into the account. Four years after this, he deposits a further R6 000 into the account. The interest rate for the first 4 years is 13% per annum compounded semi-annually. For the next 3 years, the interest rate increases to 14% per annum compounded quarterly. Calculate the future value of the savings at the end of the seven year period.

Kwv 3

It is the 1st January 2007. Leah decides to deposit R2 000 into a savings account at the end of the year. At the end of 2008, she deposits double her first amount into the account. At the end of the 2009, she deposits double her second amount into the account. The interest rate for the first two years (2007-2008) is 9% per annum compounded monthly. The interest rate for the remaining two years (2009-2010) is 8% per annum compounded monthly. Calculate the value of her investment at the end of 2010.

Kwv 4

Joseph deposits Rx into a savings account. Two years later, he deposits a further $R2x$ into the account. Three years after this, he deposits $R3x$ into the account. The interest rate for the five years is 18% per annum compounded monthly. He receives R60 000 at the end of the five year period. Calculate the value of x

5. Nominal and annual effective rates

- **Conversion between annual effective and nominal interest rate:**

Note:

- If you have calculated the effective interest rate then there is no need of putting the value of m (compounding period) in the formula

Example 1

R40 000 is invested for five years at 16% per annum compounded monthly.

- calculate the future value of the investment using the nominal rate.
- convert the nominal rate of 16% per annum compounded monthly to the equivalent effective rate.
- now use the annual effective rate to show that the same accumulated amount will be obtained as when using the nominal rate.

Example 2

Jabu invests a certain sum of money for 5 years. She receives interest of 12% per annum compounded monthly for the first two years. The interest rate changes to 14% per annum compounded semi-annually for the remaining term. The money grows to R75 000 at the end of the 5-year period.

- Calculate the effective interest rate per annum during the first year.
- Calculate how much money Jabu invested initially.

- **Conversion between effective compounding and nominal compounding rate:**

Example 1

Change a nominal rate of 14% p.a. compounded weekly to an equivalent effective monthly rate.

Example 2

Change a nominal monthly rate of 16% p.a. compounded monthly to an equivalent effective semi-annual rate.

Example 3

R3500 is invested at 14,4% p.a. compounded quarterly. After 6 months, R1 000 is added to the investment, and the amount is reinvested at 16% p.a. compounded monthly. Find the accumulated amount after five years.

Kwv 1

Convert the following nominal rates to equivalent effective rates:

- (a) 14% per annum compounded half yearly.
- (b) 16% per annum compounded quarterly.
- (c) 12% per annum compounded monthly.
- (d) 10% per annum compounded daily.

Kwv 2

- (a) Convert an effective rate of 14,5% per annum, to a nominal rate per annum compounded half yearly.
- (b) Convert an effective rate of 13,2% per annum, to a nominal rate per annum compounded quarterly.
- (c) Convert an effective rate of 10,5% per annum, to a nominal rate per annum compounded monthly.

Kwv 3

A man invests R24 000 at 16% per annum compounded quarterly for a period of 12 years.

- (a) Calculate the future value of the investment using the nominal rate.
- (b) Convert the nominal rate of 16% per annum compounded quarterly to the equivalent effective rate (annual).
- (c) Now use the annual effective rate to show that the same accumulated amount will be obtained as when using the monthly rate.

Kwv 4

Sambulo deposited R500 000 into a fixed deposit savings account for a period of six years. The accumulated amount at the end of the six year period is R650000. Calculate the interest rate paid in each of the following cases:

- (a) The annual effective rate.
- (b) The nominal rate per annum compounded monthly.

GRADE 12**1. CALCULATING THE VALUE OF n**

Calculating number of periods (n) if:

- The money double / triple the initial value (appreciation)
- The money becomes 20% / quarter of its original value (depreciation)

➤ APPRECIATION

Example 1

Simpfiwe invests R3 000 in an account paying 8% per annum compounded annually. How long will it take for the investment to double?

Example 2

How long will it take for an amount of R50 000 to triple if the interest rate is 18% per annum compounded monthly?

Kwv 1

Alicia deposits R9 000 in a savings account. Calculate how long it will take her to double her money if the interest rate is:

- (a) 6% per annum compounded annually.
- (b) 6% per annum compounded quarterly.
- (c) 6% per annum compounded half-yearly.
- (d) 6% per annum compounded monthly.
- (e) 6% per annum simple interest.

Kwv 2

Find the time taken for a certain sum of money to double if the interest rate is 11,2% per annum compounded semi-annually.

Kwv 3

Stho opens an account at Dynamic Fashions International and spends R5 000. The interest rate charged is 24% per annum compounded monthly. How long will it take her to owe the company R8 000, if she makes no prior repayments on the account?

Kwv 4

Determine how many years it will take an investment of R2 000 to earn R1 920 in interest, if the investment was made at an interest rate of 13% per annum compounded monthly.

➤ DEPRECIATION

Example 1

A motor car costing R200 000 depreciated at a rate of 8% per annum on the reducing balance method. Calculate how long it took for the car to depreciate to a value of R90 000 under these conditions.

Example 2

Determine how long, in years, it will take for the value of a motor vehicle to decrease to 25% of its original value if the rate of depreciation, based on the reducing balance method, is 21% per annum.

Kwv 1

The computers in a school's computer centre originally cost R70 000. The computers were sold for R30 000 a few years later. Calculate how long it took the computers to depreciate to R30 000 if the rate of depreciation was:

- (a) 12% per annum reducing balance depreciation.
- (b) 12% per annum reducing balance depreciation, but calculated on a quarterly basis.
- (c) 12% per annum linear depreciation.

Kwv 2

A DVD player costing R6 000 depreciates at 9% per annum on the reducing balance scale to R4 872,10 over a period of time. What will the value of the DVD player be over the same time period, if depreciation took place on a straight-line basis at 9% per annum?

2. ANNUITIES

It deals with regular fixed payments

Two types of annuities

a) Ordinary annuity

Payments are made at the end of each period

b) Annuity due

Payments are made at the beginning of each period (immediately).

3. FUTURE VALUE ANNUITY

It work forward the end of the time line

➤ **Ordinary annuity:**

The first payment is made at the end of the month and the last payment at the end of n months

➤ **Annuity due:**

The first payment is made immediately and last payment is made at the beginning of the last period

Note:

✓ **Ordinary:**

- Payments are made in one month's time
- Starting at the end of the first month

n+1

- Payments start immediately and end on the last day

✓ **Annuity due:**

- Lat payment into account is due one month before the money become matured
- Payment made immediately and ending one month before

In a future value annuity, money is invested at regular intervals in order to save money for the future.

Suppose that at the beginning of the month, R1 000 is deposited into a bank. At the end of the month a further R1 000 is deposited and a further R1 000 at the end of the next month. If the interest rate is 6% per annum compounded monthly, how much will have been saved at the end of the second month?

Example 1

Suppose that at the beginning of the month, R1 000 is deposited into a bank. At the end of the month a further R1 000 is deposited and a further R1 000 at the end of the next month. If the interest rate is 6% per annum compounded monthly, how much will have been saved after 8 years?

Example 2

Khanyisile has just turned twenty years old and has a dream of saving R10 000 000 by the time she reaches the age of 50. She starts to pay equal monthly amounts into a retirement annuity which pays 8% per annum compounded monthly. Her first payment start on her 20th birthday and her last payment is made on her 50th birthday. How much will she pay each month?

Example 3

Thami decided to start saving money for a period of 8 years starting on 31st December 2009. At the end of January 2010 (in one month's time), he deposited R2300 into the savings plan. Thereafter, he continued making deposits of R2300 at the end of each month for the planned 8 year period. The interest rate remained fixed at 10% per annum compounded monthly. How much will he have saved at the end of his 8 year plan which started on the 31st December 2009?

Kwv 1

Habib decides to save money for ten years in a Unit Trust fund. He immediately deposits R800 into a savings account. Thereafter, at the end of each month, he deposits R800 into the fund and continues to do this for the ten-year period. Interest is 15% p.a. compounded monthly. Calculate the final value of this investment.

Kwv 2

Alida decides to start saving for a car. On her 16th birthday, she deposits R5 000 into a bank account with an interest rate of 18% p.a. compounded quarterly. She continues to make quarterly payments of R5 000 until the last payment on her 24th birthday. How much money will she then have at her disposal to finance the purchase of a new car?

Kwv 3

John decides to invest money into the share market in order to become a millionaire in ten years time. He believes that he can average a return of 25% p.a. compounded monthly. In one month's time, he wishes to start making monthly payments into an account. How much must he invest per month in order to obtain his R1 000 000?

Kwv 4

Adolene wants to save up R250 000 in 5 years' time in order to purchase a car. She starts making monthly payments into an account paying 13% per annum compounded monthly, starting immediately. How much will she pay each month?

Kwv 5

Lebogang deposits R5 000 into an account paying 14% per annum compounded half-yearly. Six months later, she deposits R400 into the account. Six months after this, she deposits a further R400 into the account. She then continues to make half-yearly deposits of R400 into the account for a further nine years. Calculate the value of her savings at the end of the savings period.

4. FURTHER ACCUMULATED AMOUNT

Note:

- If the payments stop for certain periods and the money then remain in the account to accumulate for further period:
- Add the increasing factor

Example 1

Lungisani decides to open savings account for his baby daughter's future education. On opening the account, he immediately deposits R2000 into the account and continues to make monthly payments at the end of each month thereafter for a period of 16 years. The interest rate remains fixed at 15% per annum compounded monthly.

a) how much money will he have accumulated at the end of the 16th year?

b) at the end of the 16-year period, he leaves the money in the account for a further year. How much money will he then have accumulated?

Example 2

In order to supplement his state pension after retirement, a school teacher aged 30 takes out a retirement annuity. He makes monthly payments of R1000 into the fund and the payments start immediately. The payments are made in advance, which means that the last payment of R1000 is made one month before the annuity pays out. The interest rate for the annuity is 12% per annum compounded monthly. Calculate the future value of the annuity in 25 years time.

Example 3

Sly deposits R7000 into an account paying 14% per annum compounded half-yearly. Six months later, she deposits R400 into the account. Six months after this, she deposits a further R400 into the account. She then continues to make half-yearly deposits of R400 into the account for a period of nine years from the deposit of R400. Calculate the value of her savings at the end of the savings period.

5. STARTING LATER AND ENDING EARLY TO PAY

Note:

The amount must be forwarded up to the start of payments

The amount will accumulate for the remaining periods

Example 1

Mzamo pays R3000 into a saving account at the end of each month starting in 3 months time. The interest is 18% per annum compounded monthly. He pays his final R3000 six months before the time he wishes to withdraw the money. If the investment period, starting from now, is 8 years, calculate the future value of the investment at the end of the 8 year.

6. INFLATION ON THE EQUAL PAYMENTS

Note:

The amount will accumulate after the change of the interest rate.

Example 1

Sipho opens a savings account and immediately deposits R1000 into the account. He continues to make monthly payments of R1000 into the account for a period of 3 years. The interest rate for the first year is 18% per annum compounded monthly. Thereafter, the interest rate changes to 19% per annum compounded monthly for the next two years. Calculate the value of his investment at the end of the savings period.

7. MISSING THE CERTAIN PAYMENT(S)

Note:

The amount will accumulate

Example 1

Wilmoth starts a five year savings plan. At the beginning of the month he deposits R2000 into the account and makes a further deposit of R2000 at the end of that month. He then continues to make month end payments of R2000 into the account for the five year period (starting from his deposit). The interest rate is 6% per annum compounded monthly.

a) calculate the future value of his investment at the end of the 5 year period.

b) due to financial difficulty, Wilmoth misses the last two payments of R2000. What will the value of his investment now be at the end of the 5 year period?

Example 2

Sandile opens a savings account and immediately deposits R1000 into the account. He continues to make monthly payments of R1000 into the account for a period of one year. The interest rate remains fixed at 18% per annum compounded monthly. Due to financial difficulty, he missed the 4th payment. Calculate the future value of his investment.

Example 3

A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.

- a) Show that the loan would be paid off in 234 months.
- b) Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120th, 121st, 122nd and 123rd months. At the end of the 124th month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments. Calculate the value of this new instalment.

8. SINKING FUND

A **sinking fund** is an investment that is made to replace expensive equipment / items in a few years' time. It is used as a "savings account" that will accumulate funds over a period of time, which will enable the investor to purchase expensive items or to fund expensive capital outlays in a few years' time.

The steps:

- Calculate the replacement/expected/new cost (appreciation)
Formula:.....or
- Calculate the scrap value/trade-in/resale/decay value (depreciation)
Formula:.....or
- Sinking(f_v) = new cost – scrap value
Formula:.....

The monthly instalments in the sinking fund:

Use the future value annuity formula.

Formula:.....

9. SERVICES TO BE MADE

Talking some money for service of the equipment:

The sinking fund will not only lose the future value, it will also lose the interest earned on the future value at the end of that particular period. In order to calculate the future value of the amount, you first need to get the annual effective rate and then use the future value annuity formula.

Therefore new sinking fund = future value – future value of the withdrawals.

Equal instalment must be increased so as sinking fund cannot be affected.

Example 1

Antony's small business called Postal Emporium purchase a photocopying machine for R200 000. The photocopy machine depreciates in value at 20% per annum on a reducing balance. Antony's business wants to buy a new machine in 5 years time. A new machine will cost much more in the future and its cost will escalate at 16% per annum effective. The old machine will be sold at scrap value after 5 years. A sinking is set up immediately in order to save up for the new machine. The proceeds from the sale of the old machine will be used together with the sinking fund to buy the new machine.

The small business will pay equal monthly amounts into the sinking fund and interest earned is 18% per annum compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the five year period.

- a) find the scrap value of the old machine of the old machine.
- b) find the cost of the new machine in five years time.
- c) find the amount required in the sinking fund in five years time.
- d) find the equal monthly payments made into the sinking fund.
- e) suppose that the business decides to service the machine at the end of each year for the five year period. R3000 will be withdrawn from the sinking fund at the end of each year starting one year after the original machine was bought.
 - 1) calculate the reduced value of the sinking fund at the end of the five year period due to these withdrawals.
 - 2) calculate the increased monthly payment into the sinking fund which will yield the original sinking fund amount as well as allow for withdrawals from the fund for the services.

Example 2

A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

a) The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay?

b) • One month after purchasing his present tractor, the farmer deposits x rands into an account that pays interest at a rate of 12% p.a. compounded monthly.

• He continued to deposit the same amount at the end of each month for a total of 60 months.

• At the end of 60 months he has exactly the amount that is needed to purchase a new tractor, after he trades in his present tractor. Calculate the value of x .

c) Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will the new monthly deposit be?

Example 3

A printing press currently costs R850 000. The value of the machine is expected to drop at a rate of 7% p.a. simple interest, whilst the cost of a replacement machine escalates at a rate of 14% p.a. compounded annually. The press is expected to have a useful lifetime of 8 years.

a) calculate the scrap value of the old machine.

b) calculate the cost of the replacement machine.

c) calculate the amount needed to replace the old machine, if the scrap value is used as part of the payment for the new machine.

d) A sinking fund is set up to provide for this balance, paying interest at 15% p.a. compounded monthly. Determine the monthly amount that should be paid into the sinking fund to realize this. Payments start immediately and end 6 months before replacement.

Example 4

A bus company owns vehicles to the value of R1 500 000.

a) The vehicles are depreciating at a rate of 18% per annum on a reducing balance. Calculate their market value after 5 years.

b) The average rate of inflation over the next five years is expected to be 4,5% per annum and management expects to sell the old vehicles for their market value in 5 years' time. Calculate the value of the sinking fund which will be necessary to buy brand new vehicles after 5 years.

c) What monthly payments will the management of the bus company need to make in order to achieve this target if they are offered an interest rate of 6% per annum, compounded monthly, on their savings?

Kwv 1

A printing press is bought for R140 000. The cost of a new press is expected to rise by 18% p.a. while the rate of depreciation is 20% p.a. on the reducing balance. The life span of the press is 6 years.

- (a) Find the scrap value of the old press.
- (b) Find the cost of a new press in 6 year's time.
- (c) Find the value of the sinking fund that will be required to purchase the new press in six year's time, if the proceeds from the sale of the old press (at scrap value) will be utilised.
- (d) The company sets up a sinking fund to pay for the new press. Payments are to be made into an account paying 13,2% p.a. compounded monthly. Find the monthly payments, if they are to commence one month after the purchase of the old press and cease at the end of the six year period.

Kwv 2

A company bought a large generator for R227 851. It depreciates at 23% p.a. on a reducing-balance. A new machine is expected to appreciate in value at a rate of 17% p.a. A new machine will be purchased in five years time.

- (a) Find the scrap value of the old machine in five years from now.
- (b) Find the cost of a new machine in five years from now.
- (c) The company will use the money received from the sale of the old machine (at scrap value) as part payment for the new one. The rest of the money will come from a sinking fund that was set up when the old machine was bought. Monthly payments, which started one month after the purchase of the old machine, have been paid into a sinking fund account paying 11,4% p.a. compounded monthly. The payments will finish three months before the purchase of the new machine. Calculate the monthly payments into the sinking fund that will provide the required money for the purchasing of the new machine.

10. PRESENT VALUE ANNUITIES(LOAN)

It works backwards the beginning of the time line.

- **Hire purchase loans**

In a hire purchase loan, the interest payable is calculated at the start of the loan using the formula for simple interest. The total interest on the loan must be paid in full. This means that if you want to pay off the loan earlier than the loan period, you will still need to pay the full amount of interest owed. The monthly payments are calculated by dividing the loan amount by the number of months. You are required to pay all of these amounts and there is no advantage in paying off the loan early.

- **Reducing balance loans**

In bank loans, interest is paid on the reducing balance. The lower the balance, the less interest you have to pay. The advantage of these loans is that any additional payments into the loan, you are able to reduce the duration of the loan as well as save a lot of interest.

- **The present value annuity**

A reducing balance loan is often referred to as a present value annuity. In a present value annuity, a sum of money is normally borrowed from a financial institution and paid back with interest by means of regular payments at equal interval over a time period. The loan is said to be amortised (paid off) when it together with interest charges paid off. The interest is calculated on the reducing balance.

Note:

- The first payment is made at the end of the month and the last payment at the end of n months, so this is an ordinary annuity.
- There is no need of using present value formula if we have the future value of the loan.
- If we are given a deposit, this money is taken off from the value of the loan:

$$\text{Loan} = \text{cost} - \text{deposit}$$

- **Total amount paid off to the loan: $Tp = x.n + \text{deposit}$**

Suppose that a loan is repaid by means of a payment of R1000 one month after the loan was granted and one further payment of R 1000 one month after the first payment of R1000. Calculate the amount borrowed if the interest rate is 6% per annum.

Example 1

Scelo takes out a one-year bank loan to pay for an expensive laptop. The interest rate is 18% per annum compounded monthly and monthly repayments of R1650, 24 are made starting one month after the granting of the loan. Show that the amount scelo borrowed was R18 000.

Example 2

Suppose that 8 year loan is repaid by means of monthly payments of R1000 starting one month after the loan was granted. The interest rate was 6% per annum compounded monthly. Calculate the amount borrowed.

Example 3

David takes out a bank loan to pay for his new car. He repays the loan by means of monthly payments of R4000 for a period of five years starting one month after the granting of the loan. The interest rate is 24% per annum compounded monthly. Calculate the purchase price of his new car.

Example 4

Richards takes out a bank loan to pay for his new car. He pays an initial amount of R10 000. He then makes monthly payments for a period of five years starting one month after the granting of the loan. The interest rate is 24% per annum compounded monthly. Calculate the monthly payments if the car originally cost him R173 804, 43.

11. PERCENTAGE DEPOSITS**Note:**

- If 15% is paid as a deposit then the money to be borrowed is 85% of the cost/ calculates 15% of the money from the cost to get the money to be borrowed:

Loan=(100% – *given%deposit*). *cost* or

Loan= (*cost* – *given%*). *cost*

Example 1

Mxo plans to buy a car for R125 000, 00. He pays a deposit of 15% and takes out a bank loan for the balance. The bank charges 12, 5% p.a. compounded monthly.

Calculate:

- The value of the loan borrowed from the bank
- The monthly repayment on the car if the loan is repaid over 6 years

Example 5

BMW inherits R400 000 from his father. He invests the money at an interest rate of 12% per annum compounded monthly. He wishes to earn a monthly salary from the investment for a period of twenty years starting in one month's time. How much will he receive each month?

Example 6

Tumelo takes out a retirement annuity that will supplement his pension when he retires, thirty years' time. He estimates that he will need R2, 5 million in this retirement fund at that stage. The interest rate he earns is 7% per annum compounded monthly.

a) calculate his monthly payment into this fund if he starts paying immediately and makes his final payment in 30 year's time

b) the retirement fund does not pay out the R2, 5 million when Tumelo retires. Instead he will be paid monthly amounts, for a period of twenty years, starting one month after his retirement. If the interest that he earns over this period is calculated at 7% per annum compounded monthly, determine the monthly payments he will receive.

Example 7

Bhekani takes out a twenty year loan of R100 000. She repays the loan by means of equal monthly payment starting three months after the granting of the loan. The interest rate is 18% per annum compounded monthly. Calculate the monthly payments.

Kwv 1

How much can be borrowed from a bank if the borrower repays the loan by means of 30 equal monthly payments of R1 250, starting in one month's time, if interest is 12% p.a. compounded monthly?

Kwv 2

How much can be borrowed from a bank if the borrower repays the loan by means of equal quarterly payments of R2 000, starting in three months time? The interest rate is 18% p.a. compounded quarterly and the duration of the loan is ten years.

Kwv 3

Twenty-five semi-annual payments are made, starting six months from now, in order to repay a loan of R100 000. What is the value of each payment if interest is 18,6% p.a. compounded semi-annually?

Kwv 4

What amount must be invested now in order for the investor to receive equal payments of R2 000 per month from the bank for 3 years, starting in one month's time? Interest is 18% p.a. compounded monthly.

Kwv 5

John inherits R1 000 000 from his late father. He invests the money at an interest rate of 14% per annum compounded monthly. He wishes to earn a monthly salary from the investment for a period of 20 years starting one month from now. How much will he receive each month?

Kwv 6

Michael takes out a retirement annuity that will supplement his pension when he retires, thirty years from now. He estimates that he will need R2,5 million in this retirement fund at that stage. The interest rate he earns is 9% per annum compounded monthly.

(a) Calculate his monthly payment into this fund if he starts paying immediately and makes his final payment in 30 years' time.

(b) The retirement fund does not pay out the R2,5 million when Mark retires.

Instead he will be paid monthly amounts, for a period of 20 years, starting one month after his retirement. If the interest rate that he earns over this period is calculated at 7% per annum compounded monthly, determine the monthly payments he will receive.

12. OUTSTANDING BALANCE ON A LOAN**Two methods**

a) **Balance on a loan = (loan + interest) - (instalments + interest)**

We need to move the original amount of the loan forward with interest and also the regular payment that were made

b) **Present value formula**

This method just focuses on the payments that still have to be made and work backwards on the time line.

Note:

- Outstanding balance on any loan is always calculated directly after the last payment is made.

Example 1

Themba takes out a one-year bank loan of R18 000 to pay for an expensive laptop. The interest rate is 18% per annum compounded monthly and monthly repayments of R165, 24 are made starting one month after the granting of the loan.

- a) calculate his balance outstanding after he has paid the sixth instalment.
- b) calculate his balance outstanding after he has the ninth instalment.

Example 2

Jabulani takes out a bank loan for R250 000. The interest rate charged by the bank is 18,5% per annum compounded monthly.

- a) What will his monthly repayment be if he pays the loan back over five years, starting four months after the granting of the loan?
- b) Calculate the balance outstanding after the 25th repayment.

13. INFLATION BASED ON THE OUTSTANDING BALANCE**Note;**

- After calculating the outstanding balance on a loan for the certain period of time, if then interest changes;

Use the balance on the loan in order to calculate the monthly instalment after the change.

Example 3

Mr KWV has just finished paying off his twenty-year home loan of 400 000. During the first 5 years the interest rate was 24% per annum compounded monthly. Thereafter, and for the rest of the term, the interest rate decreased to 18% per annum compounded monthly.

- a) calculate his initial monthly payment.
- b) calculate his balance outstanding at the end of December in 5th year.
- c) when the interest rate changed after 5 years, Mr KWV was able to pay a decreased monthly payment starting at the end of January in the 6th year. Calculate what this new repayment was.

Example 4

A car that costs R130 000 is advertised in the following way: 'No deposit necessary and first payment due three months after date of purchase.' The interest rate quoted is 18% p.a. compounded monthly.

- a) Calculate the amount owing two months after the purchase date, which is one month before the first monthly payment is due.
- b) Herschel bought this car on 1 March 2009 and made his first payment on 1 June 2009. Thereafter he made another 53 equal payments on the first day of each month.
- (i) Calculate his monthly repayments.
- (ii) Calculate the total of all Herschel's repayments.
- c) Hashim also bought a car for R130 000. He also took out a loan for R130 000, at an interest rate of 18% p.a. compounded monthly. He also made 54 equal payments. However, he started payments one month after the purchase of the car. Calculate the total of all Hashim's repayments.
- d) Calculate the difference between Herschel's and Hashim's total repayments.

14. CALCULATING THE NUMBER OF PAYMENTS ON FUTURE VALUE

Note:

Knowledge of logarithms must be also applied

Example 1

It is the 31st December 2010. Anna decides to start saving money and wants to save R300 000 by paying monthly amounts of R4000, starting in one month's time (on 31 January 2011), into a savings account paying 15% p.a. compounded monthly. How long will it take Anna to accumulate the R300 000? The duration of the loan starts on the 31st December 2010, even though the first payment is not made on the 31 Dec 2010.

Example 2

It is 31st December 2010. Anna decides to start saving money and wants to save R300 000 by paying monthly amounts of R4000 starting immediately (on 31 December 2010), into a savings account paying 15% p.a. compounded monthly. How long will it take Anna to accumulate the R300 000? The duration of the loan starts on the 31st December 2010.

Kwv 1

R500 is invested each month, starting in one month's time, into an account paying 15% p.a. compounded monthly. How long will it take to accumulate R10 000?

Kwv 2

R2 000 is deposited into a savings account. Six months later, a further R2 000 is deposited into the account. Thereafter, amounts of R2 000 are deposited every six months into the savings account. The interest rate for the savings account is 16% p.a. compounded semi-annually. How long will it take to accumulate R100 000?

Kwv 3

Neeran opens a savings account and immediately deposits R1 000 into the account. He continues to make monthly payments of R1 000 into the account for a period of three years. The interest rate for the first year is 12% per annum compounded monthly. Thereafter, the interest rate changes to 15% per annum compounded monthly for the next two years. Calculate the value of his investment at the end of the savings period.

15. CALCULATING NUMBER OF PAYMENT ON LOANS

Note:

Knowledge of logarithms must be also applied

Example 1

Khangelani borrows R500 000 from a bank and repays the loan by means of monthly payments of R8 000, starting one month after the granting of the loan. Interest is fixed at 18% per annum compounded monthly.

- a) how many payments of R8000 will be made and what will the final payment be?
- b) how long is the loan period?

Example 2

Khetha buys a future to the value of R10 000. He borrows the money on 1 February 2010 from a financial institution that charges interest at a rate of 9,5% p.a. compounded monthly. Khetha agrees to pay monthly instalments of R450. The agreement of the loan allows Khetha to start paying these equal monthly instalments from 1 August 2010.

- a) calculate the total amount owing to the financial institution on 1 July 2010
- b) how many months will it take Khetha to pay back the loan?
- c) what is the balance of the loan immediately after Khetha has made the 25th payment?

Example 3

Nokwanda receives a bursary of R80 000, 00 for her studies at university. She invests the money at a rate of 13, 75% p.a. compounded yearly. She decides to withdraw R25 000,00 at the end of each year for her studies, starting at the end of the first year. Determine for how many full years will this investment finance her studies.

Example 4

Sbu negotiates a loan of R300 000 with a bank which has to be repaid by means of monthly payments of R5 000 and a final payment which is less than R5 000. The repayments start one month after the granting of the loan. Interest is fixed at 18% per annum, compounded monthly.

- a) Determine the number of payments required to settle the loan.
- b) Calculate the balance outstanding after Jill has paid the last R5 000.
- c) Calculate the value of the final payment made by Jill to settle the loan.
- d) Calculate the total amount that Jill repaid to the bank.

16. COMPARING INVESTMENTS AND LOANS**Example 1**

Chief wants to borrow money to buy a motorbike that costs R55 000, 00 and plans to repay the full amount over a period of 4 years in monthly instalments. He is presented with TWO options:

Option 1: The bank calculates what Andrew would owe if he borrows R55 000,00 for 4 years at a simple interest rate of 12,75% p.a., and then pays that amount back in equal monthly instalments over 4 years.

Option 2: He borrows R55 000, 00 from the bank. He pays the bank back in equal instalments over 4 years, the first payment being made at the end of the first month. Compound interest at 20% p.a. is charged on the reducing balance.

- a) If Andrew chooses Option 1, what will his monthly instalment be?
- b) Which option is the better option for Andrew? Justify your answer with appropriate calculations.
- c) What interest rate should replace 12, 75% p.a. in Option 1 so that there is no difference between the two options?

